On bounded solutions of Linear elliptic operators with measurable coefficients - De Giorgi's theorem revisted
joint work with M. Schäffner

We consider the classical framework of the famous De-Giorgi-Nash-Moser theorem:

$$\operatorname{div}(A(x)\nabla u) = f\,, \tag{0.6}$$

where $A(x)$ is a symmetric, elliptic matrix field, f is given and $u : U \subset \mathbb{R}^n \to \mathbb{R}$ is the unknown.

N. Trudinger was the first one to relax the assumptions on the coefficients matrix $A(x)$. He was able to derive boundedness results if the matrix is barley integrable in the right spaces. In particular he was able to show that if

$$\lambda(x)|\xi|^2 \leq \xi \cdot A(x)\xi \leq \Lambda(x)|\xi|^2 \quad \forall x$$

and the $\lambda^{-1} \in L^p, \Lambda \in L^q$ satisfying

$$\frac{1}{p} + \frac{1}{q} < \frac{2}{n}$$

The integrability condition had been considerably improved by P. Bella and M. Schäffner in the framework of the Moser-iteration to

$$\frac{1}{p} + \frac{1}{q} < \frac{2}{n-1}\,.$$

A counterexample had been constructed by Franchi, Serapioni, and Serra Cassano under the

$$\frac{1}{p} + \frac{1}{q} > \frac{2}{n-1}\,.$$

The aim of this talk is to revisit De Giorgi's original approach having in mind the question concerning the optimal integrability assumption on the coefficient field. We will present how this question is surprising linked to a question in linear programming with infinite horizon.

This talk will be about my ongoing project with M. Schäffner, hence about work in progess.