Discussion 5 OH 8-10 Monday

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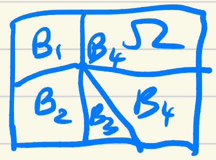
- Inclusion-exclusion formula
  $P(A \cup B \cup C)$ $P(A \cap B \cap C)$
- randomly draw with replacement
  without replacement
- independence
  conditional probability



  Bayes' formula / partition $P(A) = \sum_i P(A \cap B_i)$

$$P(B_k | A) = \frac{P(A|B_k) \cdot P(B_k)}{\sum_i P(A|B_i) \cdot P(B_i)}$$

$$= \sum_i P(A|B_i) \cdot P(B_i)$$

- CDF $\boxed{P(X \le a) = F(a)}$

$$P(X < a) = \lim_{t \to a^-} F(t) = \lim_{\substack{\varepsilon \to 0 \\ \varepsilon > 0}} F(a - \varepsilon)$$

$P(a < X < b)$
$= P(a \le X \le b)$ if X continuous

		$E[X]$	$\mathbb{E}[g(X)]$
p.m.f	$P(X=k) = p(k)$	$\sum_k k\, p(k)$	$\sum_k g(k) \cdot p(k)$
p.d.f.	$\int_a^b f(x)dx = P(a \le X \le b) = P(a < X < b)$	$\int_{-\infty}^{+\infty} x f(x) dx$	$\int_{-\infty}^{+\infty} g(x) f(x) dx$

- $Var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \ge 0$

- Gaussian random variable $Z \sim N(0, 1)$ (0: mean, 1: variance)

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad x \in \mathbb{R}$$

$$\mathbb{E}[Z^{2k+1}] = 0, \quad k \in \mathbb{N}$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$v = -e^{-x^2/2}$$

$$dv = x e^{-x^2/2}$$

$$\mathbb{E}[Z^{2k}] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x^{2k} \cdot e^{-x^2/2}\,dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \boxed{x^{2k-1}} \cdot \boxed{x e^{-x^2/2}\,dx}$$

$$= \frac{1}{\sqrt{2\pi}} x^{2k-1} \left(-e^{-x^2/2}\right)\Big|_{-\infty}^{+\infty} - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (2k-1) \cdot x^{2k-2} \left(-e^{-x^2/2}\right) dx$$

$$\to 0$$

$$= \frac{(2k-1)}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x^{2k-2} e^{-x^2/2}\,dx = (2k-1) \cdot \mathbb{E}[Z^{2k-2}]$$

$$= (2k-1) \cdot (2k-3) \cdot \mathbb{E}[Z^{2k-4}]$$

$$\mathbb{E}[Z^{2k}] = (2k-1)!!$$

mean → variance

Fact 3.61. Let μ be real, $\sigma > 0$, and suppose $X \sim \mathcal{N}(\mu, \sigma^2)$. Let $a \neq 0$, b real, and $Y = aX + b$. Then $Y \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$, that is, Y is normally distributed with mean $a\mu + b$ and variance $a^2\sigma^2$.

In particular, $Z = \frac{X-\mu}{\sigma}$ is a standard normal random variable.

$$X \sim N(\mu, \sigma^2)$$

$$P(c \le X \le d)$$

$$= P(c < X < d)$$

$$= P(c - \mu < X - \mu < d - \mu)$$

$$= P\left(\frac{c-\mu}{\sigma} < \boxed{\frac{X-\mu}{\sigma}} < \frac{d-\mu}{\sigma}\right)$$

$$= \int_{\frac{c-\mu}{\sigma}}^{\frac{d-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx$$

$$\frac{X-\mu}{\sigma} \sim N(0,1)$$

σ → standard deviation

- Approximation $X \sim Bin(n,p)$ by $\boxed{Y \sim N(np,\ np(1-p))}$

if $\boxed{np(1-p) > 10}$

$$P(X=c)$$
$$= P(c-\tfrac{1}{2} < X < c+\tfrac{1}{2})$$
$$\approx P(c-\tfrac{1}{2} < Y < c+\tfrac{1}{2})$$
$$= P\left(\frac{c-\frac{1}{2}-np}{\sqrt{np(1-p)}} < \boxed{\frac{Y-np}{\sqrt{np(1-p)}}} < \frac{c+\frac{1}{2}-np}{\sqrt{np(1-p)}}\right)$$

$$Z \sim N(0,1)$$

- Approximation $X \sim Bin(n,p)$

by $Y \sim Poisson(np)$

if $\boxed{np^2 \text{ small.}}$