Discussion 3

Zhichao Wang OH: 8–10 AM Mon

HSS 5072

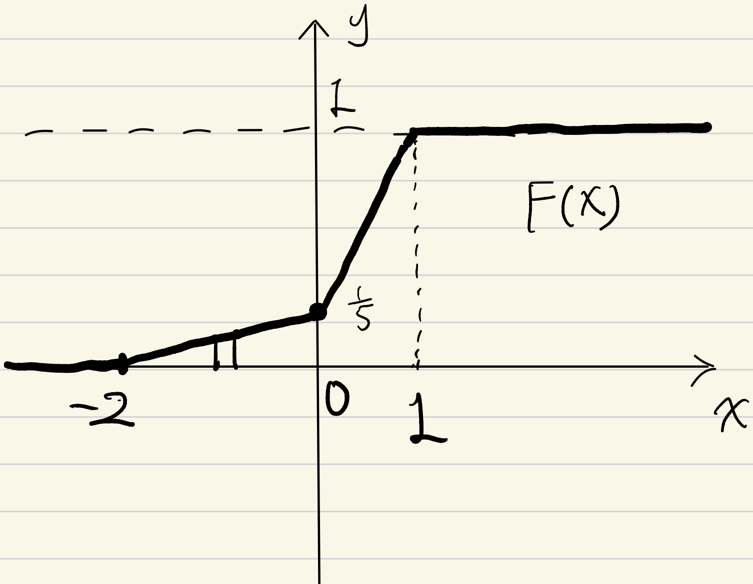
zhw036@ucsd.edu

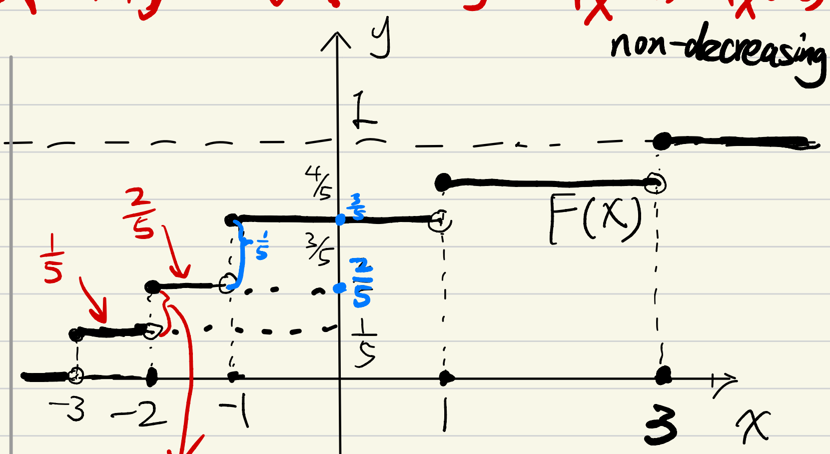
Cumulative Distribution function (CDF)

$$F_X(x) = P(X \le x) \qquad x \in \mathbb{R}$$

$x_1 \le x_2 \quad \{X \le x_1\} \subseteq \{X \le x_2\} \Rightarrow F_X(x_1) \le F_X(x_2)$

non-decreasing





$P(X=-2) = \frac{1}{5}$

Continuous r.v.

$\Downarrow$

p.d.f $f(x)$

prob. density function

$$\boxed{f(x) = F_X'(x)}$$

$f(x) \ge 0 \qquad \int_{-\infty}^{+\infty} f(x)\,dx = 1$

$P(X=a) = 0$

Discrete r.v

$\Downarrow$

p.m.f

mass

$P(X=k) =$ size of jump at k.

$\sum_k P(X=k) = 1$

① Uniform random variable

$[1,2] \cup [3,4]$ uniformly randomly select a number X

$1 \le X \le 2$ or $3 \le X \le 4$

$F_X(t) = P(X \le t)$

if $t < 1$ $F_X(t) = 0$

if $1 \le t \le 2$ $F_X(t) = P(1 \le X \le t) = \frac{t-1}{2}$

if $2 < t < 3$ $F_X(t) = P(X \le t)$

$= P(1 \le X \le 2) = \frac{1}{2}$

if $3 \le t \le 4$ $F_X(t) = \frac{t-3+1}{2} = \frac{t-2}{2}$

if $t > 4$ $F_X(t) = 1$

$1 \le t \le 2 \Rightarrow f_X(t) = F_X'(t) = \frac{1}{2}$

$2 < t < 3 \Longrightarrow f_X(t) = F_X'(t) = 0$

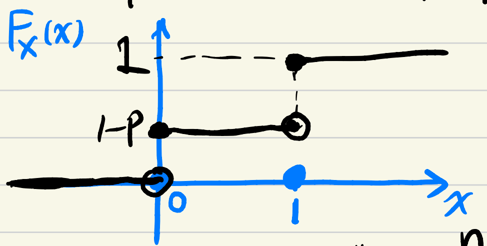
$3 \le t \le 4 \Rightarrow f_X(t) = \frac{1}{2}$

$t < 1,\ t > 4 \Rightarrow f_X(t) = 0$

② Bernoulli **random variable** $p \in (0,1)$

Ber(p) $X = \begin{cases} 0, & \text{with probability } 1-p \\ 1, & \text{with probability } p. \end{cases}$

$$P(X=0) = 1-p, \quad P(X=1) = p$$



③ $1 = (\underbrace{p}_{=a} + \underbrace{(1-p)}_{=b})^n = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}$

Binomial **random variable**

Bin(n, p)

$$X = \begin{cases} 0, & P(X=0) = (1-p)^n \\ 1, & P(X=1) = np(1-p)^{n-1} \\ \vdots & \\ k, & \boxed{P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}} \\ \vdots & \\ n, & P(X=n) = p^n \end{cases}$$

EX: $X=$ Number of correct guesses at 5 true-false questions when you randomly guess all answers.

$X = 0, 1, 2, \cdots, 5$ $\quad n=5 \quad p=\frac{1}{2}$

$Bin(5, \frac{1}{2})$

$$P(X=0) = \left(\frac{1}{2}\right)^5$$

$$P(X=2) = \binom{5}{2}\left(\frac{1}{2}\right)^5$$

$\boxed{F}$ $\boxed{T}$ $\boxed{F}$ $\boxed{T}$ $\boxed{F}$

- Number of winning lottery tickets when you buy 10 tickets of the same kind
- Number of left-handers in a randomly selected sample of 100 unrelated people

EX: X = # number of rolls of a 10-sided fair die until a 3 or 5 or 7 appears

find p.m.f. of X

$X = 1, 2, 3, \ldots$

$k \in \mathbb{N}$

$$P(X=k) = \left(\frac{7}{10}\right)^{k-1} \cdot \left(\frac{3}{10}\right)$$

$$\boxed{\sum_{k=1}^{+\infty} P(X=k) = 1}$$

EX 3.40. Give a p.m.f or p.d.f. of some variable that has C.D.F. $F(x)$ that satisfies $\boxed{F(n) = 1 - \frac{1}{n}}$ $n \in \mathbb{Z}_+$.

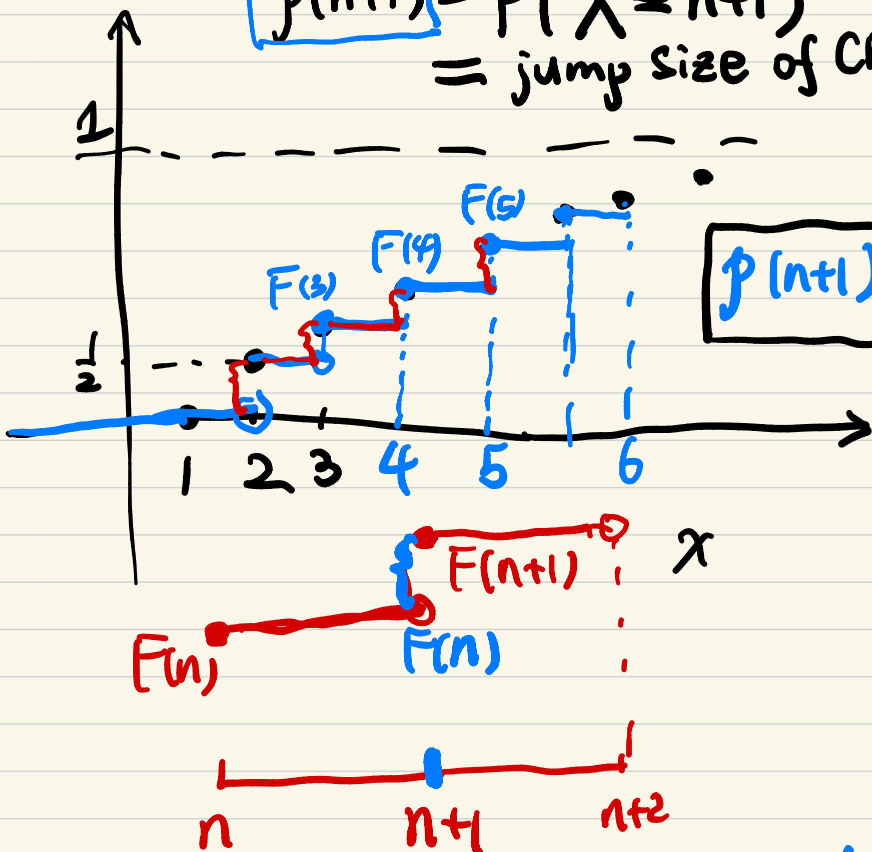
EX 3.40. Give p.m.f or p.d.f. of some variable that has C.D.F. $F(x)$ that satisfies $\boxed{F(n) = 1 - \frac{1}{n}}$ $n \in \mathbb{Z}_+$.

$\boxed{p(n+1)} = P(X = n+1)$

$= $ jump size of CDF at $n+1$

discrete random variable

p.m.f.



$\boxed{p(n+1)} = F(n+1) - F(n)$

$= 1 - \frac{1}{n+1} - \left(1 - \frac{1}{n}\right)$

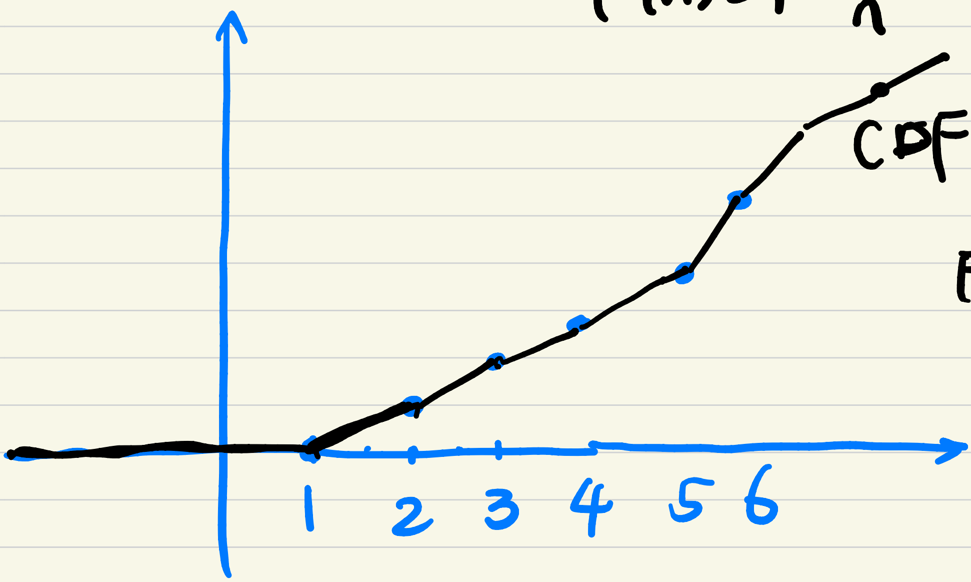
$= \frac{1}{n} - \frac{1}{n+1}$

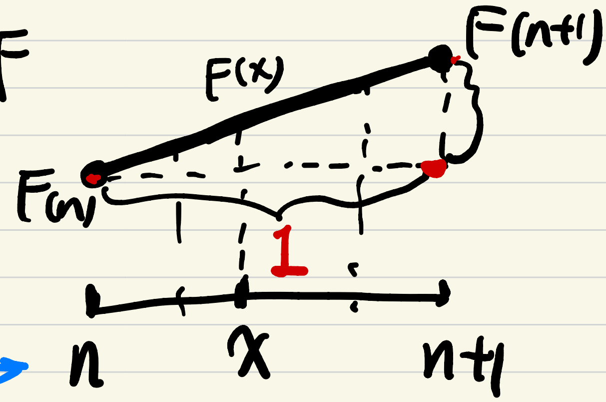
$= \frac{1}{n(n+1)}$

p.m.f. $p(n+1) = \frac{1}{n(n+1)}$ $n \in \mathbb{Z}_+$

$F(n) = 1 - \frac{1}{n} \quad n \in \mathbb{Z}_+ \qquad F(x)$

$F(n) = 1 - \frac{1}{n}$



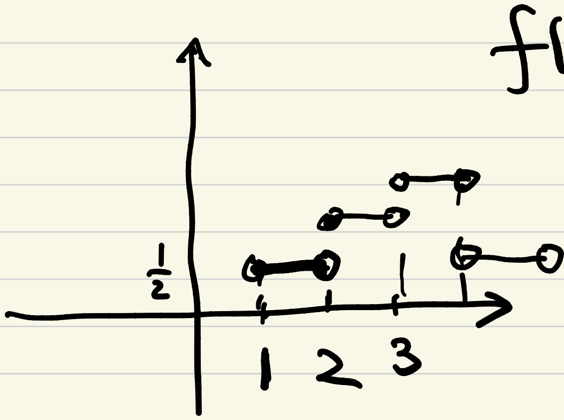


$F'(x) = f(x) : \text{pdf}$

$$\frac{1-\frac{1}{n+1}-\left(1-\frac{1}{n}\right)}{1} = \frac{F(n+1)-F(n)}{(n+1)-n} = \text{slope} \qquad x \in (n, n+1)$$

$$= \frac{1}{n(n+1)}$$

$$f(x) = \frac{1}{n(n+1)}, \quad n < x < n+1, \quad n \in \mathbb{Z}_+$$



$$\cancel{1} - \frac{1}{n+1} - \cancel{1} + \frac{1}{n}$$

$$= \frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)}$$