Discussion 2: OH: 8 – 10 AM Mon

zhw036@ucsd.edu   HSS 5072

- Inclusion-exclusion formulas:

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$$

- conditional prob.

$$P(C\mid D) = \frac{P(C\cap D)}{P(D)} = \frac{P(CD)}{P(D)} \quad (P(D)\neq 0)$$

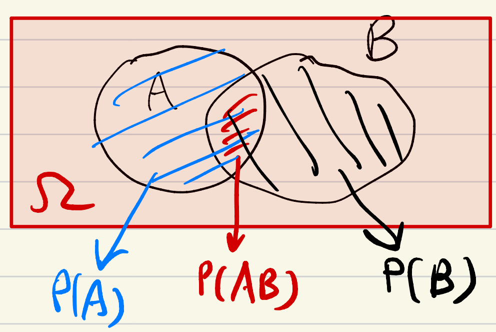
$$P(CD) = P(C\mid D)\cdot P(D) \qquad P(D\mid D) = 1$$

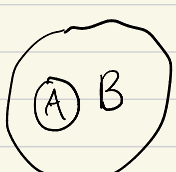
- independence between A & B

$$P(A\mid B) = P(A)$$

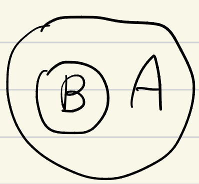
$$P(AB) = P(A)\,P(B)$$

events $A \neq \emptyset$, $B \neq \emptyset$ $\subseteq \Omega$





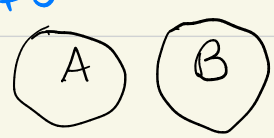
$(A \subseteq B)$

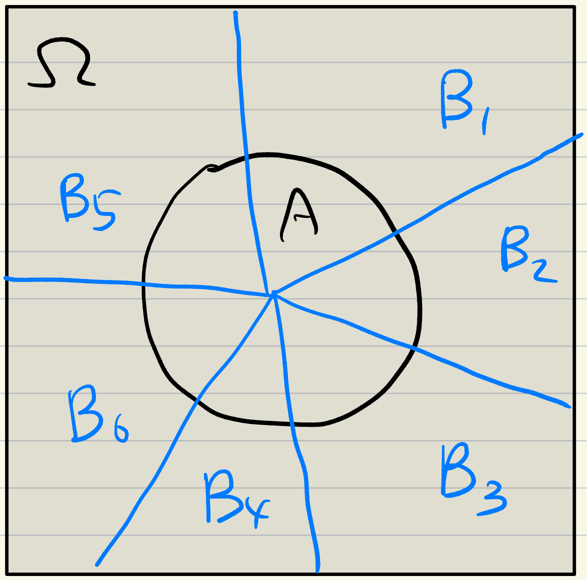


$$P(A) = P(AB) \overset{?}{\neq} P(A)\cdot P(B)$$

$$0 = P(AB) \neq P(A)\cdot P(B)$$

if $A \subseteq B$, $P(B) \neq 1$, $P(A) \neq 0$ $\Big\} \Rightarrow$ A & B not indep.





$A \cap B = \emptyset$, $P(A) \neq 0$, $P(B) \neq 0$ $\Rightarrow$ A & B not indep.

$$P(\Omega) = \sum_{i=1}^{n} P(B_i)$$

$$\bigcup_{i=1}^{n} B_i = \Omega, \quad B_i \cap B_j = \phi \Rightarrow \{B_i\}_{i=1}^{n} \text{ partition of } \Omega$$

$$P(A) = \sum_{i=1}^{n} P(A \cap B_i)$$

$$= \sum_{i=1}^{n} P(AB_i) = \sum_{i=1}^{n} P(A \mid B_i) P(B_i)$$

$$P(B_k \mid A) = \frac{P(B_k A)}{P(A)} = \frac{P(A \mid B_k) \cdot P(B_k)}{\sum_{i=1}^{n} P(A \mid B_i) P(B_i)}$$

$1 \leq k \leq n$

$$B_k A = B_k \cap A$$

$$= A \cap B_k = AB_k$$

EX 1.40: 1 green ball, 1 red ball, 1 yellow ball, 1 white ball

draw 4 balls with replacement

GYYR

GGYY

What is the probability that there exists at least one color that is repeated exactly twice?

① # outcomes: $|\Omega| = 4^4$

$G = \{$**green** ball appears exactly twice$\}$

$P(G) = \dfrac{\binom{4}{2}\cdot 3\cdot 3}{4^4}$

G G $\boxed{\cdot\cdot}$ $\boxed{\cdot\cdot}$

$P(G\cap Y\cap R) = 0$ $\quad P(G\cap Y\cap R\cap W) = 0$

there exists at least one color appears exactly twice

$$P(G\cup Y\cup R\cup W) = P(G)+P(Y)+P(R)+P(W)$$
$$- P(G\cap Y) - P(G\cap R) - P(G\cap W)$$
$$- P(Y\cap R) - P(Y\cap W) - P(R\cap W)$$
$$= 4P(G) - 6P(G\cap Y)$$

$$P(G) = \frac{\binom{4}{2}\cdot 3\cdot 3}{4^4} = \frac{6\times 9}{4^4}$$

$$P(G\cap Y) = \frac{\binom{4}{2}}{4^4}$$

GGYY

② at least one color appears twice

GGYR

only one color repeats twice

GGYY

2 color repeat twice respectively

$$P(\text{only one color repeats exactly twice}) = \frac{\binom{4}{1}\cdot\binom{4}{2}\cdot 3\cdot 2}{4^4}$$

$$P(\text{2 colors appears twice}) = \frac{\binom{4}{2}\binom{4}{2}}{4^4}$$

③ Consider complement

P(there is no color appears exactly twice)

= P(each color appears exactly 1 time) GRYW

+P(some color appears 3 times) GGGY

+P(1 color appears 4 times) GGGG

$$= \frac{4!}{4^4} + \frac{\binom{4}{1}\binom{4}{1}\cdot 3}{4^4} + \frac{4}{4^4}$$

EX 2.10 :

3 fair dice
- 4-sided {1, 2, 3, 4}
- 6-sided {1, 2, 3, 4, 5, 6}
- 12-sided {1, 2, 3, 4, ······, 12}

We randomly pick one and then roll it to get a number

- What is probability that we get "4" given that we get a 6-sided die at beginning?

$P(A|B_1)$

- If in the end, we get "4" what is the probability I pulled out a 6-sided die ?

$P(B_1|A)$

$A = \{$ the outcome of the roll is 4 $\}$

$B_1 = \{$ we get a 6-sided die at the begining $\}$

$$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$$

$$P(A|B_1) = \frac{1}{6} \qquad P(A|B_2) = \frac{1}{4}$$

$$P(A|B_3) = \frac{1}{12}$$

$$P(B_1|A) = \frac{P(A|B_1)\,P(B_1)}{\sum_{i=1}^{3} P(A|B_i)\cdot P(B_i)}$$

Monty Hall problem

$\boxed{p}$ $\boxed{e}$ $\boxed{e}$

1 2 3

assume prize in door 1.

first selection		host selection		if we switch
1 ($\frac{1}{3}$)	p	2	$\frac{1}{3}p$	no prize
	$1-p$	3	$\frac{1}{3}(1-p)$	no prize
2 ($\frac{1}{3}$)		3	$\frac{1}{3}$	prize
3 ($\frac{1}{3}$)		2	$\frac{1}{3}$	prize

p(switching is good) $= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

i.e. you switch and get prize

p(switching is bad) $= \frac{1}{3}p + \frac{1}{3}(1-p) = \frac{1}{3}$.