Name (last, first): ____________________________________________

Student ID: ________________________________________________

☐ Write your name and PID on the top of EVERY PAGE.

☐ Write the solutions to each problem on separate pages. CLEARLY INDICATE on the top of each page the number of the corresponding problem. Different parts of the same problem can be written on the same page (for example, part (a) and part (b)).

☐ The exam consists of 4 questions. Your answers must be carefully justified to receive credit.

☐ This exam will be scanned. Make sure you write ALL SOLUTIONS on the paper provided. DO NOT REMOVE ANY OF THE PAGES.

☐ No calculators, phones, or other electronic devices are allowed.

☐ Remember this exam is graded by a human being. Write your solutions NEATLY AND COHERENTLY, or they risk not receiving full credit.

☐ You are allowed to use one 8.5 by 11 inch sheet of paper with handwritten notes (on both sides); no other notes (or books) are allowed.

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1. (20 points) Let $(\Omega, \mathcal{F}, P)$ be a probability space.

   (a) Suppose that $A, B \in \mathcal{F}$ and $P(A) = 0.6$, $P(B) = 0.9$. Show that
   
   $$0.5 \leq P(A \cap B) \leq 0.6.$$

   (b) Show that for any $F, G \in \mathcal{F}$

   $$P(F \cap G) \geq 1 - P(F^C) - P(G^C).$$
(ADDITIONAL SPACE FOR WORK, clearly INDICATE the problem you are working on)
2. (20 points) An urn contains 2 white balls and 4 black balls. You remove the balls one by one from the urn (without replacement).

(a) What is the probability that the first two balls removed from the urn are black?
(b) What is the probability that the last removed ball is white?
(ADDITIONAL SPACE FOR WORK, clearly INDICATE the problem you are working on)
3. (20 points) Consider a point \( P = (X, Y) \) chosen uniformly at random inside of the triangle in \( \mathbb{R}^2 \) that has vertices \((1, 0), (0, 1), \) and \((0, 0)\). Let \( Z = \max(X, Y) \) be the random variable defined as the maximum of the two coordinates of the point. For example, if \( P = (\frac{1}{2}, \frac{1}{3}) \), then \( Z = \max(X, Y) = \frac{1}{2} \).

Determine the cumulative distribution function of \( Z \).

Determine if \( Z \) is a continuous random variable, a discrete random variable, or neither. If continuous, determine the probability density function of \( Z \). If discrete, determine the probability mass function of \( Z \). If neither, explain why.

(Hint: Draw a picture.)
(ADDITIONAL SPACE FOR WORK, clearly INDICATE the problem you are working on)
4. (20 points) You roll three fair four-sided dice.

(a) (15 points) Compute the probability that there will be at least one four, given that all three dice give different numbers.

(b) (15 points) Compute the (unconditional) probability that there will be at least one four. [Hint. Use complement]