1. Prove that for any $n \in \mathbb{N}$
   \[(2n)! < 2^{2n}(n!)^2.\]

2. Prove that the set
   \[S := \left\{ \frac{n}{n+3}(2 + (-1)^n) : n \in \mathbb{N} \right\}\]
is bounded. Determine $\text{sup } S$ and $\text{inf } S$ (provide the proof).

3. By checking the definition of a convergent sequence, compute the limit of the sequence
   \[(a_n)_{n=1}^{\infty} \text{ with } a_n = \sqrt{n + 1} - \sqrt{n}.\]

4. Determine
   \[\lim_{n \to \infty} \left( \frac{1}{2n} - \frac{2}{2n} + \frac{3}{2n} - \cdots + \frac{2n-1}{2n} - \frac{2n}{2n} \right).\]

5. Prove that the sequence $(b_n)_{n=1}^{\infty}$ with
   \[b_n = 1 + \frac{1}{2 \cdot 2} + \frac{1}{3 \cdot 2^2} + \frac{1}{4 \cdot 2^3} + \cdots + \frac{1}{n \cdot 2^{n-1}}\]
is convergent.