1. (5 points) Let \( O, P, Q, R \) be points in \( \mathbb{R}^3 \) with coordinates
\[
O = (0, 0, 0), \quad P = (3, 2, -1), \quad Q = (0, -1, 2), \quad R = (6, 2, 0).
\]
Which of the following vectors is parallel to the vector \( \mathbf{v} = \langle -1, \frac{1}{3}, \frac{1}{2} \rangle \)?

A. \( \overrightarrow{PQ} \)
B. \( \overrightarrow{OP} \)
C. \( \overrightarrow{OQ} \)
D. \( \overrightarrow{PR} \)
E.* None of the above

2. (5 points) Find a unit vector \( \mathbf{u} \) in the direction opposite of \( \langle 2, -2, 1 \rangle \).

A. \( \mathbf{u} = \langle -\frac{4}{\sqrt{3}}, -\frac{2}{\sqrt{3}}, \frac{4}{\sqrt{3}} \rangle \)
B.* \( \mathbf{u} = \langle -\frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \rangle \)
C. \( \mathbf{u} = \langle 1, \frac{1}{2}, -1 \rangle \)
D. \( \mathbf{u} = \langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \rangle \)
E. None of the above

3. (5 points) Assume that \( \mathbf{u} \cdot \mathbf{v} = -1, \|\mathbf{u}\| = 2, \|\mathbf{v}\| = 2 \). What is the value of \( (\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + 3\mathbf{v}) \)?

A. -3
B. * 12
C. -6
D. 0
E. None of the above

4. (5 points) Compute the area of the triangle with vertices
\[
P = (1, 1, 0), \quad Q = (2, 3, 0), \quad R = (2, 2, -1).
\]

A. \( \sqrt{3} \)
B. -1
C. \( \frac{1}{2} \)
D.* \( \frac{\sqrt{6}}{2} \)
E. None of the above
5. (5 points) Find the equation of the plane which passes through point \( P = (-1, 1, -1) \) and is parallel to \( x + 2y + \frac{1}{2}z = -1 \).

A. * \(-2x - 4y - z = -1\)
B. \( x + \frac{1}{2}y + 2z = -1\)
C. \( x + 2y + z = 0\)
D. \(-x - y - z = 1\)
E. None of the above

6. (5 points) Find the distance from the point \((0, -1, 0)\) to the plane \(-2x - y + 3z = 3\).

A. \(\frac{\sqrt{7}}{2}\)
B. \(\frac{2}{\sqrt{3}}\)
C. \(\frac{\sqrt{7}}{\sqrt{3}}\)
D. * \(\frac{2}{\sqrt{14}}\)
E. None of the above

7. (5 points) Suppose \( \mathbf{r}(t) = (e^{-t^3}, \cos(\pi t), 2t - 1) \) represent the position of a particle at time \( t \), where the \( z \)-component represents the height of the particle. What is the velocity of the particle when its height is 5?

A. * \((-1, 0, 2)\)
B. \((3e^{-3}, -\pi, 2)\)
C. \((e^3, 0, 5)\)
D. \((1, \pi, 2)\)
E. None of the above

8. (5 points) Find the parametric equation of the tangent line to \( f(t) = (-t^3, 2t^3, 2t - 4) \) at \( t_0 = 2 \).

A. \((-8, 16, 0) + t(16, -32, -4)\)
B. \((16, 12, 1) + t(16, 12, 1)\)
C. * \((16, -32, -4) + t(-12, 24, 2)\)
D. \((16, 8, 1) + t(-12, 24, 2)\)
E. None of the above
9. (5 points) Let \( f(x, y) = e^{x^2 - 2x + y^2} \). Compute the partial derivative \( f_x(2, 0) \).

A. \( 2 \)  
B. \( -2e^2 \)  
C. \( e^2 \)  
D. 0  
E. None of the above

10. (5 points) Let \( f(x, y) = (x^2 + y)e^{2y} \). Compute \( f_{xy}(1, 1) \).

A. \( 4e^2 \)  
B. 2  
C. \( 2e \)  
D. 0  
E. None of the above

11. (5 points) Given \( z = f(x, y) \), \( x = x(u, v) \), \( y = y(u, v) \), with \( x(2, 1) = 1 \) and \( y(2, 1) = 2 \), calculate \( z_u(2, 1) \) using the values below

\[
\begin{align*}
f_x(2, 1) &= c, & f_y(2, 1) &= d, & f_x(1, 2) &= p, & f_y(1, 2) &= q, \\
x_u(2, 1) &= 1, & x_v(2, 1) &= 2, & y_u(2, 1) &= 3, & y_v(2, 1) &= -1.
\end{align*}
\]

A. \( -p + q \)  
B. \( 3c + 2d \)  
C. \( p + q + 2 \)  
D. \( p + 3q \)  
E. None of the above

12. (5 points) Calculate the directional derivative of \( f(x, y) = x^2y^2 \) in the direction of \( \mathbf{v} = (2, 1) \) at the point \( P = (1, -1) \). Remember to normalize the direction vector.

A. \( \frac{\sqrt{5}}{12} \)  
B. \( \frac{2}{\sqrt{5}} \)  
C. \( -\frac{3}{\sqrt{5}} \)  
D. 0  
E. None of the above
13. (5 points) Find the critical point of the function \( f(x,y) = 3x^2 + 3y^2 + xy + 7x \).

A. \((-1,0)\)
B. \(\left(\frac{1}{2}, \frac{3}{2}\right)\)
C. \((1,6)\)
D.* \(\left(-\frac{6}{5}, \frac{1}{5}\right)\)
E. None of the above

14. (5 points) The point \( P = (-2,0) \) is a critical point of the function \( f(x,y) = x^3 - y^2 - 12x \).

Use the second derivative test to determine if \( P \) is a point of local minimum, local maximum or a saddle point.

A. Local minimum
B.* Local maximum
C. Saddle point
D. The second derivative test is inconclusive
E. None of the above

15. (5 points) Find the minimum value of the function \( f(x,y) = x^2 + y^2 \) subject to the constraint \( 4x + 2y = 5 \).

A. \(\frac{10}{36}\)
B.* \(\frac{5}{4}\)
C. \(\frac{120}{25}\)
D. Minimum does not exist
E. None of the above

16. (5 points) Find three positive real numbers whose sum is 3 and whose product is a maximum.

A.* 1
B. \(\frac{1}{3}\)
C. 3
D. Maximum does not exist
E. None of the above