MATH 180A PRACTICE MIDTERM 1

Name (last, first): _____

Student ID: _____

☐ **Write your name and PID on the top of EVERY PAGE.**

☐ **Write the solutions to each problem on separate pages. CLEARLY INDICATE on the top of each page the number of the corresponding problem. Different parts of the same problem can be written on the same page (for example, part (a) and part (b)).**

☐ **The exam consists of 4 questions. Your answers must be carefully justified to receive credit.**

☐ **This exam will be scanned. Make sure you write ALL SOLUTIONS on the paper provided. DO NOT REMOVE ANY OF THE PAGES.**

☐ **No calculators, phones, or other electronic devices are allowed.**

☐ **Remember this exam is graded by a human being. Write your solutions NEATLY AND COHERENTLY, or they risk not receiving full credit.**

☐ **You are allowed to use one 8.5 by 11 inch sheet of paper with handwritten notes (on both sides); no other notes (or books) are allowed.**

1. Let $(\Omega, \mathcal{F}, P)$ be a probability space.
   (a) Suppose that $A, B \in \mathcal{F}$ satisfy
   $$P(A) + P(B) > 1.$$
   Making no further assumptions on A and B, prove that $A \cap B \neq \emptyset$.
   (b) Prove that A is independent from itself if and only if $P(A) \in \{0, 1\}$.

2. An urn contains 2 white balls and 4 black balls. You remove the balls one by one from the urn (without replacement).
   (a) What is the probability that the first two balls removed from the urn are black?
   (b) What is the probability that the last removed ball is white?

3. A box contains 3 coins, two of which are fair and the third has probability 3/4 of coming up heads. A coin is chosen randomly from the box and tossed 3 times.
   (a) What is the probability that all 3 tosses are heads?
   (b) Given that all three tosses are heads, what is the probability that the biased coin was chosen?

4. You roll three fair *four-sided* dice.
   (a) Compute the probability that there will be at least one four, *given* that all three dice give different numbers.
   (b) Compute the (unconditional) probability that there will be at least one four. [**Hint.** Use complement]