Name (last, first): _____

Student ID: _____

☐ **Write your name and PID on the top of EVERY PAGE.**

☐ **Write the solutions to each problem on separate pages. CLEARLY INDICATE on the top of each page the number of the corresponding problem. Different parts of the same problem can be written on the same page (for example, part (a) and part (b)).**

☐ **The exam consists of 4 questions. Your answers must be carefully justified to receive credit.**

☐ **This exam will be scanned. Make sure you write ALL SOLUTIONS on the paper provided. DO NOT REMOVE ANY OF THE PAGES.**

☐ **No calculators, phones, or other electronic devices are allowed.**

☐ **Remember this exam is graded by a human being. Write your solutions NEATLY AND COHERENTLY, or they risk not receiving full credit.**

☐ **You are allowed to use one 8.5 by 11 inch sheet of paper with handwritten notes (on both sides); no other notes (or books) are allowed.**

1. (20 points) The position of an object is given by

$$\mathbf{r}(t) = \langle e^{t^2-2t+1}, \cos(\pi t), 3t^2 - 6t \rangle.$$

(a) Compute $\mathbf{v}(t)$, the velocity of the object.

(b) Determine the time $t_0 > 0$ at which $\mathbf{v}(t_0) = \vec{0}$.

(c) Determine the position of the object at time t_0 (when the object's speed is zero).

(ADDITIONAL SPACE FOR WORK, clearly INDICATE the problem you are working on)

2. (20 points) Find the equation of the tangent plane to the surface determined by the function

$$f(x, y) = \sqrt{xy^2 + x + y}$$

at the point $(1, 0)$.

(ADDITIONAL SPACE FOR WORK, clearly INDICATE the problem you are working on)

3. (20 points) Let

$$f(x, y) = (x + y)e^{xy^2}.$$

(a) Find the gradient of the function f at the point $(0, 0)$. Find the directional derivative of the function f at the point $(0, 0)$ in the direction $\vec{v} = \langle 1, -1 \rangle$. (Hint. Do not forget to normalize $\vec{v}$.)

(b) Find the unit vector in the direction of the maximal rate of increase for the function f at the point $(0, 0)$. What is the value of the directional derivative in this direction?

(ADDITIONAL SPACE FOR WORK, clearly INDICATE the problem you are working on)

4. (20 points) Use the chain rule to find the partial derivative $\dfrac{\partial z}{\partial u}$ for

$$z = x^2(1+y^2)^2,$$

where

$$x = u + 2v, \quad y = uv.$$

You may leave your answer as a product of terms, but your answer should not have any derivative operations remaining to be performed. Your final answer should only be a function of u and v.

(ADDITIONAL SPACE FOR WORK, clearly INDICATE the problem you are working on)

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