Name (last, first): ______________________________

Student ID: ______________________________

☐ **Write your name and PID on the top of EVERY PAGE.**

☐ **The exam consists of 16 questions. Each question has only one correct answer. Be sure to completely fill in the appropriate bubble in the bubble answer sheet.**

☐ **DO NOT REMOVE ANY OF THE PAGES.**

☐ **No calculators, phones, or other electronic devices are allowed.**

☐ **You are allowed to use one 8.5 by 11 inch sheet of paper with handwritten notes (on both sides); no other notes (or books) are allowed.**

1. (5 points) Let O, P, Q, R be points in $\mathbb{R}^3$ with coordinates

$$O = (0,0,0), \quad P = (-1,1,0), \quad Q = (2,-5,2), \quad R = (-2,1,-2).$$

Which of the following vectors is parallel to the vector $\mathbf{v} = \left\langle \frac{-1}{2}, 1, \frac{-1}{3} \right\rangle$?

A. $\overrightarrow{QR}$
B. $\overrightarrow{OQ}$
C. $\overrightarrow{OP}$
D.* $\overrightarrow{PQ}$
E. None of the above

2. (5 points) Find a *unit* vector $\mathbf{u}$ in the direction opposite of $\langle -2, 4, 4 \rangle$.

A. $\mathbf{u} = \left\langle -1, \frac{-1}{2}, 1 \right\rangle$
B. $\mathbf{u} = \left\langle 1, \frac{1}{2}, -1 \right\rangle$
C.* $\mathbf{u} = \left\langle \frac{1}{3}, \frac{-2}{3}, \frac{-2}{3} \right\rangle$
D. $\mathbf{u} = \left\langle \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} \right\rangle$
E. None of the above

3. (5 points) Assume that $\mathbf{u} \cdot \mathbf{v} = 3$, $\|\mathbf{u}\| = 4$, $\|\mathbf{v}\| = 1$. What is the value of $(\mathbf{u} - 4\mathbf{v}) \cdot (\mathbf{u} + \mathbf{v})$?

A.* 3
B. -2
C. 1
D. 26
E. None of the above

4. (5 points) Compute the area of the triangle with vertices

$$P = (0,1,0), \quad Q = (2,2,0), \quad R = (1,0,1).$$

A.* $\frac{\sqrt{14}}{2}$
B. $2\sqrt{7}$
C. $\frac{1}{2}$
D. $\sqrt{14}$
E. None of the above

5. (5 points) Find the equation of the plane which passes through point $P = \left(\frac{1}{2}, 1, -1\right)$ and is parallel to $x - y + z = -2$.

   A. $x + z = -\frac{1}{2}$
   B. $2x - 2y + 2z = 0$
   C.* $-2x + 2y - 2z = 3$
   D. $2x + 2y + 2z = 1$
   E. None of the above

6. (5 points) Find the distance from the point $(0, 1, 0)$ to the plane $-2x - y + 3z = 3$.

   A. $\dfrac{\sqrt{14}}{2}$
   B. $\dfrac{3}{\sqrt{3}}$
   C. $\dfrac{2}{\sqrt{3}}$
   D.* $\dfrac{4}{\sqrt{14}}$
   E. None of the above

7. (5 points) Suppose $\mathbf{r}(t) = \langle e^{-t}, \sin(\pi t), 3t - 5\rangle$ represent the position of a particle at time t, where the z-component represents the height of the particle. What is the velocity of the particle when its height is 4?

   A. $\langle -e^{-3}, 0, 4\rangle$
   B.* $\langle -e^{-3}, -\pi, 3\rangle$
   C. $\langle 3e^{-3}, -\frac{\pi}{2}, 3\rangle$
   D. $\langle -3e^{-3}, 0, 2\rangle$
   E. None of the above

8. (5 points) Find the parametric equation of the tangent line to $f(t) = \langle -4t + 2, t^3, -t^2\rangle$ at $t_0 = -1$.

   A. $\langle 4, 4, -1\rangle + t\langle 6, -1, -1\rangle$
   B. $\langle 6, -1, -1\rangle + t\langle 6, 3, -2\rangle$
   C.* $\langle 2, 2, 1\rangle + t\langle -4, 3, 2\rangle$
   D. $\langle -4, 3, 2\rangle + t\langle -4, -1, -1\rangle$
   E. None of the above

9. (5 points) Let $f(x, y) = \sin(x^2 + 2y^2)$. Compute the partial derivative $f_y\left(0, \sqrt{\frac{\pi}{2}}\right)$.

   A.* $-4\sqrt{\frac{\pi}{2}}$
   B. -2
   C. 0
   D. $2\sqrt{\frac{\pi}{2}}$
   E. None of the above

10. (5 points) Let $f(x, y) = (x - 3y^2)e^x$. Compute $f_{yx}(1, 0)$.

   A. -6
   B. $6e$
   C. $-3e$
   D.* 0
   E. None of the above

11. (5 points) Given $z = f(x, y)$, $x = x(u, v)$, $y = y(u, v)$, with $x(2, 1) = 1$ and $y(2, 1) = 2$, calculate $z_v(2, 1)$ using the values below

$$f_x(2,1) = c, \quad f_y(2,1) = d, \quad f_x(1,2) = p, \quad f_y(1,2) = q,$$
$$x_u(2,1) = 2, \quad x_v(2,1) = 3, \quad y_u(2,1) = -1, \quad y_v(2,1) = 1.$$

   A. $-p + q$
   B. $3c - d$
   C. $p - q + 3$
   D.* $3p + q$
   E. None of the above

12. (5 points) Calculate the directional derivative of $f(x, y) = x^2y^2$ in the direction of $\mathbf{v} = \langle 1, -1 \rangle$ at the point $P = (2, 1)$. Remember to normalize the direction vector.

   A.* $-\frac{4}{\sqrt{2}}$
   B. 4
   C. $\frac{12}{\sqrt{2}}$
   D. $-\frac{2}{\sqrt{6}}$
   E. None of the above

13. (5 points) Find the critical point of the function $f(x,y) = 3x^2 + 2y^2 + 2xy + 10y$.

A. $\left(\frac{3}{10}, -\frac{1}{10}\right)$
B.* $(1, -3)$
C. $(2, 3)$
D. $\left(-\frac{1}{5}, 1\right)$
E. None of the above

14. (5 points) The point $P = (1,0)$ is a critical point of the function $f(x,y) = x^3 + y^2 - 3x$. Use the second derivative test to determine if P is a point of local minimum, local maximum or a saddle point.

A.* Local minimum
B. Local maximum
C. Saddle point
D. The second derivative test is inconclusive
E. None of the above

15. (5 points) Find the minimum value of the function $f(x,y) = 3x^2 + 2y^2$ subject to the constraint $x + y = 1$.

A. $\frac{1}{4}$
B.* $\frac{30}{25}$
C. $\frac{17}{36}$
D. Maximum does not exist
E. None of the above

16. (5 points) Find three positive real numbers whose sum is 6 and whose product is a maximum.

A.* 8
B. 27
C. 51
D. Maximum does not exist
E. None of the above