Name (last, first): ______________________________

Student ID: ______________________________

☐ **Write your name and PID on the top of EVERY PAGE.**

☐ **The exam consists of 16 questions. Each question has only one correct answer. Be sure to completely fill in the appropriate bubble in the bubble answer sheet.**

☐ **DO NOT REMOVE ANY OF THE PAGES.**

☐ **No calculators, phones, or other electronic devices are allowed.**

☐ **You are allowed to use one 8.5 by 11 inch sheet of paper with handwritten notes (on both sides); no other notes (or books) are allowed.**

1. (5 points) Let O, P, Q, R be points in $\mathbb{R}^3$ with coordinates

$$O = (0,0,0), \quad P = (0,-1,3), \quad Q = (-2,6,3), \quad R = (1,1,1).$$

Which of the following vectors is parallel to the vector $\mathbf{v} = \left\langle \frac{1}{3}, -1, \frac{-1}{2} \right\rangle$?

A. $\overrightarrow{PQ}$
B. $\overrightarrow{QR}$
C. $\overrightarrow{OP}$
D.* $\overrightarrow{OQ}$
E. None of the above

2. (5 points) Find a *unit* vector $\mathbf{u}$ in the direction opposite of $\langle -1, -2, 2 \rangle$.

A. $\mathbf{u} = \left\langle 1, \frac{1}{2}, -1 \right\rangle$
B.* $\mathbf{u} = \left\langle \frac{1}{3}, \frac{2}{3}, \frac{-2}{3} \right\rangle$
C. $\mathbf{u} = \left\langle -1, \frac{-1}{2}, 1 \right\rangle$
D. $\mathbf{u} = \left\langle \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} \right\rangle$
E. None of the above

3. (5 points) Assume that $\mathbf{u} \cdot \mathbf{v} = 3$, $\|\mathbf{u}\| = 2$, $\|\mathbf{v}\| = 2$. What is the value of $(\mathbf{u} - 2\mathbf{v}) \cdot (\mathbf{u} + \mathbf{v})$?

A.* -7
B. -12
C. 5
D. 0
E. None of the above

4. (5 points) Compute the area of the triangle with vertices

$$P = (0,0,1), \quad Q = (1,0,4), \quad R = (1,1,2).$$

A. $\sqrt{14}$
B. $2\sqrt{7}$
C. $\frac{1}{2}$
D.* $\frac{\sqrt{14}}{2}$
E. None of the above

5. (5 points) Find the equation of the plane which passes through point $P = (2, 1, 1)$ and is parallel to $x - 2y + 3z = -7$.

   A. $-6x + 4y - 2z = -3$
   B. $3x - 2y + z = 5$
   C. $-3x + 2y - z = -2$
   D.* $2x - 4y + 6z = 6$
   E. None of the above

6. (5 points) Find the distance from the point $(0, 0, 1)$ to the plane $2x - y + 3z = 2$.

   A. $\frac{\sqrt{7}}{\sqrt{3}}$
   B. $\frac{3}{\sqrt{3}}$
   C.* $\frac{1}{\sqrt{14}}$
   D. $\frac{\sqrt{14}}{2}$
   E. None of the above

7. (5 points) Suppose $\mathbf{r}(t) = \langle e^{2t-4}, \sin(\pi t), 3t\rangle$ represent the position of a particle at time t, where the z-component represents the height of the particle. What is the velocity of the particle when its height is 6?

   A. $\langle 2e^4, \pi, 0\rangle$
   B. $\langle 2e^4, 2\pi, 3\rangle$
   C.* $\langle 2, \pi, 3\rangle$
   D. $\langle 2, 0, 3\rangle$
   E. None of the above

8. (5 points) Find the parametric equation of the tangent line to $f(t) = \langle -t^3, 2t^3, t+1\rangle$ at $t_0 = -1$.

   A. $\langle -1, -2, 0\rangle + t\langle -3, 6, 0\rangle$
   B.* $\langle -2, 4, 1\rangle + t\langle -3, 6, 1\rangle$
   C. $\langle -3, 6, 0\rangle + t\langle 3, 6, 1\rangle$
   D. $\langle 3, 6, 1\rangle + t\langle -3, 6, 0\rangle$
   E. None of the above

9. (5 points) Let $f(x, y) = e^{(x-1)^2-(y-1)^2}$. Compute the partial derivative $f_x(3, 1)$.

A. $-4e^3$
B.* $4e^4$
C. 0
D. 4
E. None of the above

10. (5 points) Let $f(x, y) = e^{x^2+y^2+x}$. Compute $f_{yx}(1, 1)$.

A. 1
B. $2e$
C. e^5
D.* $6e^3$
E. None of the above

11. (5 points) Given $z = f(x, y)$, $x = x(u, v)$, $y = y(u, v)$, with $x(2, 1) = 1$ and $y(2, 1) = 2$, calculate $z_u(2, 1)$ using the values below

$$f_x(2,1) = c, \quad f_y(2,1) = d, \quad f_x(1,2) = p, \quad f_y(1,2) = q,$$
$$x_u(2,1) = 2, \quad x_v(2,1) = 3, \quad y_u(2,1) = 1, \quad y_v(2,1) = -1.$$

A. $-2p + 3q$
B. $cd - 3$
C.* $2p + q$
D. $p + 2q$
E. None of the above

12. (5 points) Calculate the directional derivative of $f(x, y) = x^2y^2$ in the direction of $\mathbf{v} = \langle 1, -2 \rangle$ at the point $P = (1, -1)$. Remember to normalize the direction vector.

A. $\frac{5}{\sqrt{2}}$
B. -1
C.* $\frac{6}{\sqrt{5}}$
D. $\frac{-1}{\sqrt{5}}$
E. None of the above

13. (5 points) Find the critical point of the function $f(x, y) = 4x^2 + 3y^2 + 6xy + x$.

A. $\left(\frac{1}{3}, \frac{2}{3}\right)$
B. $(-1, 0)$
C. $\left(-\frac{1}{3}, 2\right)$
D.* $\left(-\frac{1}{2}, \frac{1}{2}\right)$
E. None of the above

14. (5 points) The point $P = (2, 0)$ is a critical point of the function $f(x, y) = x^3 - y^2 - 12x$. Use the second derivative test to determine if P is a point of local minimum, local maximum or a saddle point.

A. Local minimum
B. Local maximum
C.* Saddle point
D. The second derivative test is inconclusive
E. None of the above

15. (5 points) Find the minimum value of the function $f(x, y) = x^2 + y^2$ subject to the constraint $2x + 4y = 5$.

A. $\frac{13}{36}$
B. 2
C.* $\frac{5}{4}$
D. Minimum does not exist
E. None of the above

16. (5 points) Find three positive real numbers whose sum is 1 and whose product is a maximum.

A.* $\frac{1}{27}$
B. 1
C. 3
D. Maximum does not exist
E. None of the above