MATH 10C: Calculus III (Lecture B00)

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Today: Equations of a plane

Next: Strang 3.1

Week 3:

- homework 3 (due Monday, October 17)
- Midterm 1: Wednesday, October 19 (vectors, dot product, cross product, equations of lines and planes)

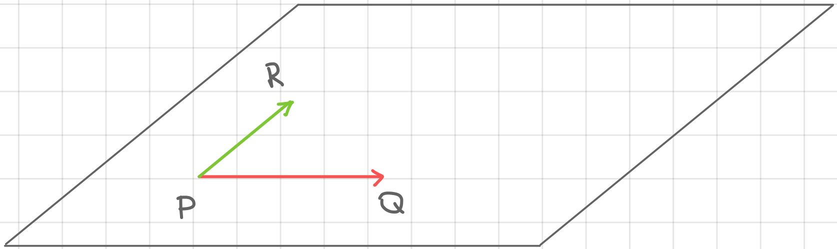
Planes

Two points determine a line: for any two points P, Q (in $\mathbb{R}^2$ or $\mathbb{R}^3$) there exists a unique line passing through P and Q. A point X is in the line through P and Q if $\overrightarrow{PX}$ is a multiple of $\overrightarrow{PQ}$, i.e., $\overrightarrow{PX} = t\,\overrightarrow{PQ}$ for some $t \in \mathbb{R}$.

Three points (that do not all lie on the same line) determine a plane: for any three points P, Q and R in $\mathbb{R}^3$ that do not all lie on the same line, there exists a unique plane that passes through these three points.

A point X is in the plane passing through P, Q and R if

Equation of a plane



Another way to describe a plane is by identifying

. If P is a point in the plane and vector $\vec{n}$ is orthogonal to the plane (called the normal vector) then point X is in this plane if and only if

Equation of a plane

Consider a plane containing point $P = (x_0, y_0, z_0)$ with normal vector $\vec{n} = \langle a, b, c \rangle$. Then point $X = (x, y, z)$ belong to this plane if and only if

(*)

If we denote , then (*) becomes

Suppose that we know the coordinates of three points P, Q, R in the plane. How can we find a normal vector to this plane?

Example

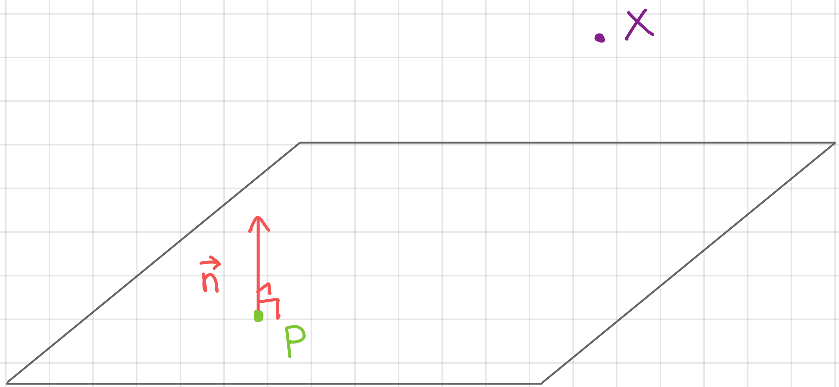
Write the vector equation for the plane containing points $P = (1,1,0)$, $Q = (-2,1,1)$, $R = (0,0,1)$

Compute the normal vector to the plane

Point $X = (x,y,z)$ is in the plane if

, or equivalently

Distance between a plane and a point

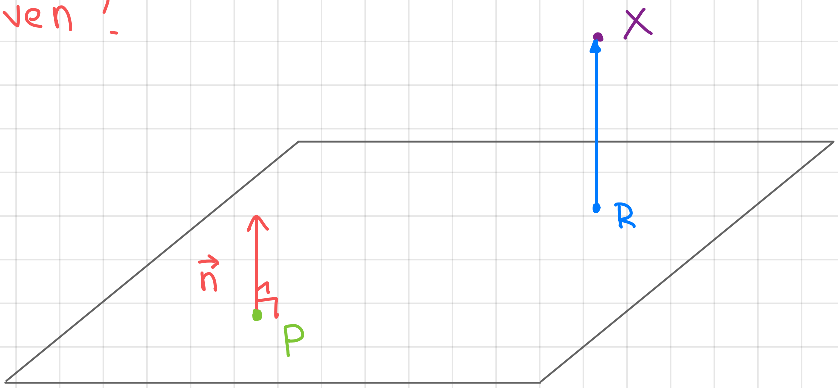


Consider a plane with point P and normal vector $\vec{n}$.
Suppose that point X does not belong to this plane.
The distance d between X and the plane is the smallest distance between X and points in the plane.
If $\overrightarrow{RX}$ is orthogonal to the plane (parallel to $\vec{n}$), then

Distance between a plane and a point

Conclusion:

How to find $\vec{RX}$ (and $\|\vec{RX}\|$) if P and $\vec{n}$ are given?



$\vec{RX} =$

Distance between a plane and a point

Example

Find the distance between the point $X = (0,0,0)$ and the plane given by $x + 2y + 3z - 3 = 0$.

This is the equation in the general form. First find the normal vector

Next we need a point in the plane (any point), i.e., any numbers x_0, y_0, z_0 such that $x_0 + 2y_0 + 3z_0 - 3 = 0$.

We can take

Then the distance from X to the plane is

Vector-valued functions

<u>Definition</u> A vector-valued function is a function that takes real numbers as inputs and gives vectors as outputs, i.e.,

$$\vec{r}(t) =$$

$$\vec{r}(t) =$$

<u>Example</u> $\vec{r}(t) =$

$\vec{r}(t) =$

<u>Remark</u> From now on we will not distinguish between the point (x, y, z) and the vector $\langle x, y, z \rangle$, both are just lists of three real numbers

Vector-valued functions

Vector valued function $\vec{r}(t)$ often represents a

Think about the motion of a planet, flight of an airplane or a bird etc.

A vector-valued function may not be defined for all real numbers. For example, $\vec{r}(t) =$

is not defined for

You can explicitly specify the set of real number for which you want to define the function by writing, e.g.,

. We call this set the

Vector-valued functions

If the domain is not explicitly specified, we assume that it is the set of

Example

$$\vec{r}(t) = \langle \frac{1}{t}, \frac{1}{\cos t}, t \rangle$$

$$\operatorname{dom}(\vec{r}(t)) =$$

Sometimes the domain is found from the problem setup. If the function describes the motion of a bird between time 0 and tim T, then the domain is