MATH 10C: Calculus III (Lecture B00)

mathweb.ucsd.edu/~ynemish/teaching/10c

Today: Method of Lagrange multipliers

Next: Review

Week 10:

- Homework 8 due Friday, December 2
- CAPES

Final: Monday, December 5, 11:30 AM - 2:30 PM

Method of Lagrange multipliers. One constraint

Problem: find the maximum/minimum of $f(x,y)$ on the curve C that is defined by the equation $g(x,y)=0$. Suppose that f is differentiable and C is smooth.

Problem solving strategy:

2. Set up the system of equations using the following template

$$\begin{cases} \nabla f(x_0,y_0) = \lambda \nabla g(x_0,y_0) \\ g(x_0,y_0) = 0 \end{cases}$$

3. Solve for x_0 and y_0 (may have multiple solutions)

4. The largest of the values of f at points (x_0,y_0) found above maximizes f on C; the smallest of the values minimizes f on C.

More about step 4

Lagrange multipliers are used to find the critical points.

The points of local minima/maxima are critical points,

but critical points are not necessarily local minima/maxima

Suppose $(x_0, y_0), \dots, (x_n, y_n)$ are the points that satisfy the Lagrange multipliers equation and $f(x_0,y_0) < f(x_1,y_1) \le \dots < f(x_n,y_n)$

- if $g(x,y)=0$ is bounded, then (x_0,y_0) minimizes f on $g(x,y)=0$, (x_n,y_n) maximizes f on $g(x,y)=0$ (we know max/min exist)
- if $g(x,y)=0$ is unbounded, visualize and determine whether f gets larger or smaller as (x,y) goes to infinity along $g(x,y)=0$
- if $g(x,y)=0$ is unbounded but we consider only a bounded part D of it, then check the value of f at the endpoints (boundary) of D

Lagrange multipliers in $\mathbb{R}^3$. Two constraints

Problem: maximize/minimize $f(x,y,z)$

subject to $g(x,y,z)=0$

$h(x,y,z)=0$

Problem solving strategy:

1. Determine the objective function f and the constraint functions g and h
2. Set up the system of equations

3. Solve the system for x_0, y_0, z_0 (may have multiple solutions)
4. Determine which of the points is max/min (if exists)

Lagrange multipliers in $\mathbb{R}^3$. Two constraints

Example Find the closest point to the origin on the line on intersection of the planes $2x+y+2z=9$, $5x+5y+7z=29$

Find the minimum of $f(x,y,z)=x^2+y^2+z^2$

subject to $2x+y+2z=9$

$5x+5y+7z=29$

1. $f(x,y,z)=x^2+y^2+z^2$,
2. Set up the system of equations:

Lagrange multipliers in $\mathbb{R}^3$. Two constraints

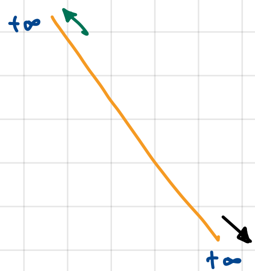
3.

Lagrange multipliers in $\mathbb{R}^3$. Two constraints

4. Min? Max?

Is the set determined by $2x+y+2z=9$ and $5x+5y+7z=29$ bounded?

How does $f(x,y,z)=x^2+y^2+z^2$ behave as (x,y,z) tends to infinity along the line?



Name (last, first): ______________________________

Student ID: ______________________________

☐ **Write your name and PID on the top of EVERY PAGE.**

☐ **The exam consists of 16 questions. Each question has only one correct answer. Be sure to completely fill in the appropriate bubble in the bubble answer sheet.**

☐ **DO NOT REMOVE ANY OF THE PAGES.**

☐ **No calculators, phones, or other electronic devices are allowed.**

☐ **You are allowed to use one 8.5 by 11 inch sheet of paper with hand-written notes (on both sides); no other notes (or books) are allowed.**

Name	**Version** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
ID	**Other**
Section	**Marking Instructions** Be sure to completely fill in the appropriate bubble. **Example** ● Ⓑ Ⓒ Ⓓ Ⓔ
Date	

	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
1	A	B	C	D	E	26	A	B	C	D	E	51	A	B	C	D	E	76	A	B	C	D	E
2	A	B	C	D	E	27	A	B	C	D	E	52	A	B	C	D	E	77	A	B	C	D	E
3	A	B	C	D	E	28	A	B	C	D	E	53	A	B	C	D	E	78	A	B	C	D	E
4	A	B	C	D	E	29	A	B	C	D	E	54	A	B	C	D	E	79	A	B	C	D	E
5	A	B	C	D	E	30	A	B	C	D	E	55	A	B	C	D	E	80	A	B	C	D	E
6	A	B	C	D	E	31	A	B	C	D	E	56	A	B	C	D	E	81	A	B	C	D	E
7	A	B	C	D	E	32	A	B	C	D	E	57	A	B	C	D	E	82	A	B	C	D	E
8	A	B	C	D	E	33	A	B	C	D	E	58	A	B	C	D	E	83	A	B	C	D	E
9	A	B	C	D	E	34	A	B	C	D	E	59	A	B	C	D	E	84	A	B	C	D	E
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12	A	B	C	D	E	37	A	B	C	D	E	62	A	B	C	D	E	87	A	B	C	D	E
13	A	B	C	D	E	38	A	B	C	D	E	63	A	B	C	D	E	88	A	B	C	D	E
14	A	B	C	D	E	39	A	B	C	D	E	64	A	B	C	D	E	89	A	B	C	D	E
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24	A	B	C	D	E	49	A	B	C	D	E	74	A	B	C	D	E	99	A	B	C	D	E
25	A	B	C	D	E	50	A	B	C	D	E	75	A	B	C	D	E	100	A	B	C	D	E

	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
101	A	B	C	D	E	126	A	B	C	D	E	151	A	B	C	D	E	176	A	B	C	D	E
102	A	B	C	D	E	127	A	B	C	D	E	152	A	B	C	D	E	177	A	B	C	D	E
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112	A	B	C	D	E	137	A	B	C	D	E	162	A	B	C	D	E	187	A	B	C	D	E
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125	A	B	C	D	E	150	A	B	C	D	E	175	A	B	C	D	E	200	A	B	C	D	E