MATH 10C: Calculus III (Lecture B00)

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Today: Method of Lagrange multipliers

Next: Strang 4.8

Week 9:

- Homework 7 due Wednesday, November 23
- Homework 8 due Friday, December 2

Method of Lagrange multipliers. One constraint

Problem: find the maximum/minimum of $f(x,y)$ on the curve C that is defined by the equation $g(x,y)=0$. Suppose that f is differentiable and C is smooth.

Problem solving strategy:

2. Set up the system of equations using the following template

$$\begin{cases} \nabla f(x_0,y_0) = \lambda \nabla g(x_0,y_0) \\ g(x_0,y_0)=0 \end{cases}$$

3. Solve for x_0 and y_0 (may have multiple solutions)

4. The largest of the values of f at points (x_0,y_0) found above maximizes f on C; the smallest of the values minimizes f on C.

More about step 4

Lagrange multipliers are used to find the critical points.

The points of local minima/maxima are critical points,

but critical points are not necessarily local minima/maxima

Suppose $(x_0,y_0),\dots,(x_n,y_n)$ are the points that satisfy the Lagrange multipliers equation and $f(x_0,y_0) \le f(x_1,y_1) \le \cdots \le f(x_n,y_n)$

- if $g(x,y)=0$ is bounded, then (x_0,y_0) minimizes f on $g(x,y)=0$, (x_n,y_n) maximizes f on $g(x,y)=0$ (we know max/min exist)
- if $g(x,y)=0$ is unbounded, visualize and determine whether f gets larger or smaller as (x,y) goes to infinity along $g(x,y)=0$
- if $g(x,y)=0$ is unbounded but we consider only a bounded part D of it, then check the value of f at the endpoints (boundary) of D

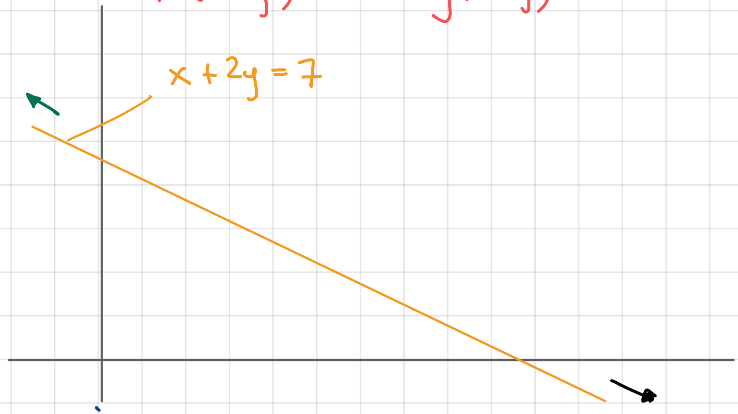
Step 4 of the last example

Example Use the method of Largange multipliers to find the minimum value of $f(x,y) = x^2 + 4y^2 - 2x + 8y$ subject to the constraint $x + 2y = 7$.

Steps 1-3: Point $(5,1)$ is the only solution to the Lagrange multipliers equation.

Step 4: What is the minimum of $f(x,y)$ on $g(x,y) = 0$?

Is $x + 2y = 7$ bounded?



Method of Lagrange multipliers

Example Maximize $f(x,y) = x(x^2+2y^2-1)$ subject to $x^2+y^2=1$.

1. Maximize $f(x,y) = x(x^2+2y^2-1)$ subject to

2.

3.

4. '

Method of Lagrange multipliers. Cobb-Douglas function

Company's production level is given by the Cobb-Douglas formula $f(x,y) = 2.5x^{0.45}y^{0.55}$, where x is the total number of labor hours, and y represents the total capital input.

Suppose 1 unit of labor costs 40 \$, one unit of capital costs 50 \$. Use the Lagrange multipliers method to find the max value of $f(x,y) = 2.5x^{0.45}y^{0.55}$ subject to budgetary constraint of 500000 \$.

1.

2. Set up the system of equations:

Method of Lagrange multipliers. Cobb-Douglas function

3. Solve the system (*):

Method of Lagrange multipliers. Cobb-Douglas function

$$x = \frac{45000}{8} = 5625 \qquad y = \frac{44000}{8} = 5500$$

4. The candidate for the maximum is $(5625, 5500)$.

Is this a maximum or a minimum?

Consider the function $2.5x^{0.45}y^{0.55}$ on the budgetary constraint line $40x + 50y = 500000$.

f can only have either one max on this line or one min on this line. Compute the value of f at the endpoints: $x=0, y=10000$ and $y=0, x=12500$

Therefore, the production is maximized with 5625 units of labor and 5500 units of capital.

Lagrange multipliers in $\mathbb{R}^3$. One constraint function

Company's production level is given by the Cobb-Douglas function $f(x,y,z) = x^{0.2}y^{0.4}z^{0.4}$, where x is labor, y is capital, z is advertizing. Suppose 1 unit of labor costs 40 \$, one unit of capital costs 50 \$, one unit of advertizing costs 100 \$. Use the Lagrange multipliers method to find the max value of $f(x,y,z)$ subject to budgetary constraints of 500000 \$.

1. Objective function: $f(x,y,z) = x^{0.2}y^{0.4}z^{0.4}$

   Constraint function: $g(x,y,z) =$

2. Compute ∇f and ∇g and set up the equations:

   $f_x =$ $\qquad$ $f_y =$ $\qquad$ $f_z =$

Lagrange multipliers in $\mathbb{R}^3$. One constraint function

Objective function: $f(x,y,z) = x^{0.2}y^{0.4}z^{0.4}$

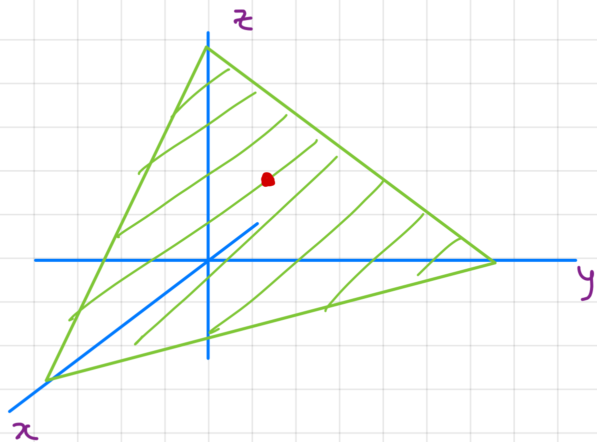
Constraint function: $g(x,y,z) = 40x + 50y + 100z - 500000 = 0$

Equations:

3. Solve the system

Lagrange multipliers in $\mathbb{R}^3$. One constraint function

Plug



4. The candidate for the maximum is

Min / max / saddle point ?