MATH 10C: Calculus III (Lecture B00)

mathweb.ucsd.edu/~ynemish/teaching/10c

Today: Absolute minima/maxima

Next: Strang 4.8

Week 8:

- Homework 7 due Wednesday, November 23

Recall: local and global minima and maxima

Def Let $z = f(x,y)$ be a function of two variables.
Then f has a local maximum at point (x_0, y_0) if

$(*)$ $f(x_0, y_0) \geq f(x,y)$ for all points (x,y) within some disk centered at (x_0, y_0). The number $f(x_0, y_0)$ is called the local maximum value. If $(*)$ holds for all (x,y) in the domain of f, we say that f has a global maximum at (x_0, y_0)

Function f has a local minimum at point (x_0, y_0) if

$(**)$ $f(x_0, y_0) \leq f(x,y)$ for all points (x,y) within some disk centered at (x_0, y_0). The number $f(x_0, y_0)$ is called the local minimum value. If $(**)$ holds for all (x,y) in the domain of f, we say that f has a global minimum at (x_0, y_0)

Local minima and local maxima are called local extrema.

Absolute (global) maxima and minima

Finding global minima/maxima for functions of one variable on a closed interval:

- find critical points
- check the critical values
- evaluate function at the endpoints of the interval.

We generalize this strategy to fuctions of two (or more) variables defined on a closed bounded set.

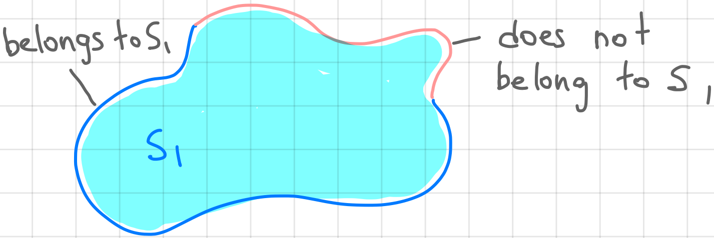
A set is called bounded if all the points of the set are

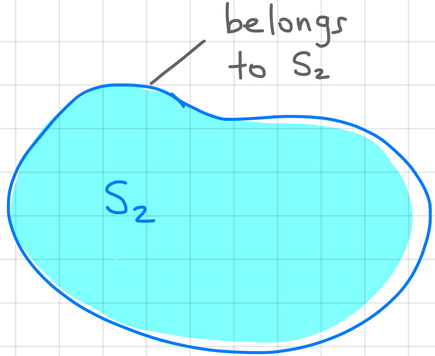
A set is called closed if it contains

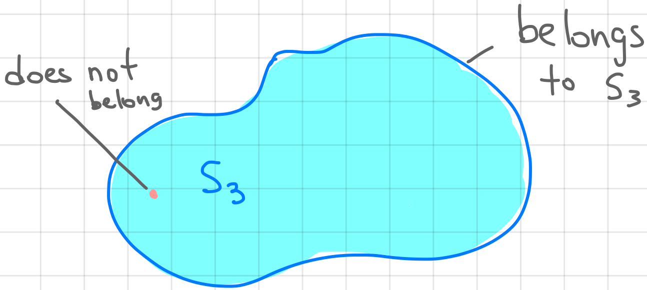
(A point P_0 is called a boundary point of a set S if any disk around P_0 contains points both inside and outside S)

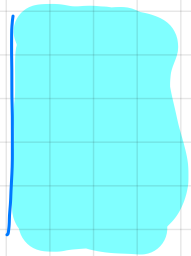
Extreme value theorem

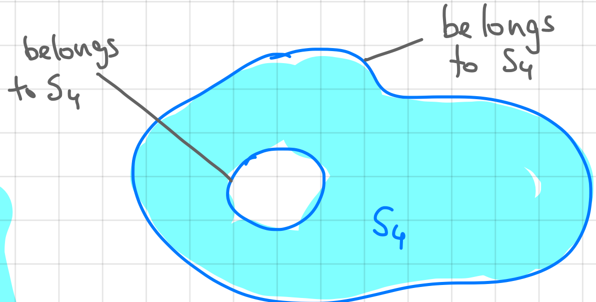
Closed sets:











<u>Theorem</u>. A continuous function f on a closed bounded set D attains an absolute maximum value at some point in D and an absolute minimum value at some point in D.

Finding absolute minima and maxima

Thm. Assume $z = f(x,y)$ is a differentiable function of two variables defined on a closed bounded set D. Then f will attain the absolute maximum value and the absolute minimum value, which are, respectively, the largest and smallest values found among the following

(i) The values of f

(ii) The values of f

Problem solving strategy for finding absolute max and min:

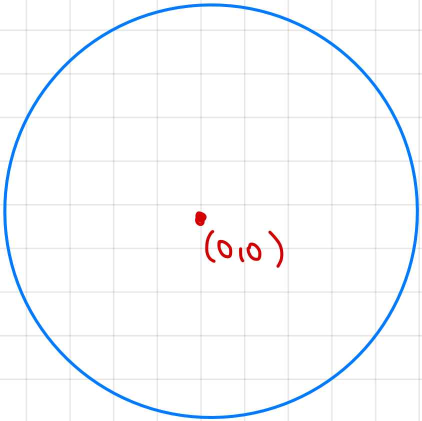
1. Determine the critical points of f in D
2. Calculate f at each of these critical points
3. Detetermine the max and min values of f on the boundary
4. Choose max/min from the values obtained in steps 2 and 3

Example

Some villagers want to set up a communication tower within 1 km of their village. They want to put it at the highest elevation possible.
Suppose that the landscape around the village is described by $f(x,y) = x(x^2+2y^2-1)$ with the village center at $(0,0)$. What is the highest point within a (horizontal) distance of 1 km from $(0,0)$?

In other words, we have to

Example



The set $D = \{(x,y) \mid x^2+y^2 \leq 1\}$ is a unit disk (including the boundary). It is a closed and bounded set.

Its boundary is a unit circle.

The maximum value can be inside the disk or on the boundary.

Remark: In general, finding the max/min value on the boundary may be nontrivial. One can

in $\mathbb{R}^2$, and find the max/min

of , where is the parametrization

of the boundary, e.g.,

Example

Step 1: Determine the critical points inside D

$$f(x,y) = x(x^2 + 2y^2 - 1)$$

$f_x =$

$f_y =$

Find the critical points by solving

Example

Check which points are in $D = \{(x,y) \mid x^2+y^2 \leq 1\}$

Step 2: Evaluate f at critical points

$$f(x,y) = x(x^2+2y^2-1)$$

Example

Step 3: Find the max value on the boundary

Parametrize the boundary

Each point on the boundary (unit circle) can be written as

so the values of f on the boundary are

We have to fint the maximum of

Example

Maximizing $\cos(t)\sin^2(t)$:

Find the critical points

Critical points are points that satisfy

Example

Compute the maximum on the boundary

$$f(x,y) = x(x^2+2y^2-1)$$

$f(1,0) =$ $\qquad\qquad$ $f(-1,0) =$

$f\left(\frac{1}{\sqrt{3}}, \pm\sqrt{\frac{2}{3}}\right) =$

$f\left(-\frac{1}{\sqrt{3}}, \pm\sqrt{\frac{2}{3}}\right) =$

Step 4: Choose the absolute maximum

Inside the disk:

On the boundary:

Conclusion: max value of f on D is $\qquad$, achieved at