MATH 10C: Calculus III (Lecture B00)

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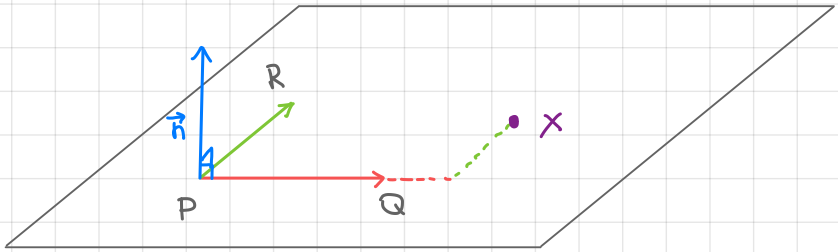
Today: Vector-valued functions

Next: Strang 3.2

Week 3:

- homework 3 (due Tuesday, October 18)
- Midterm 1: Wednesday, October 19 (vectors, dot product, cross product, equations of lines and planes)

Equation of a plane



Consider a plane containing point $P = (x_0, y_0, z_0)$ with normal vector $\vec{n} = \langle a, b, c \rangle$. Then point $X = (x, y, z)$ belong to this plane if and only if

$\vec{n} \perp \overrightarrow{PX}$, i.e. $\vec{n} \cdot \overrightarrow{PX} = 0$ vector equation of a plane

(*) $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ scalar equation of a plane

If we denote $d := -ax_0 - by_0 - cz_0$, then (*) becomes

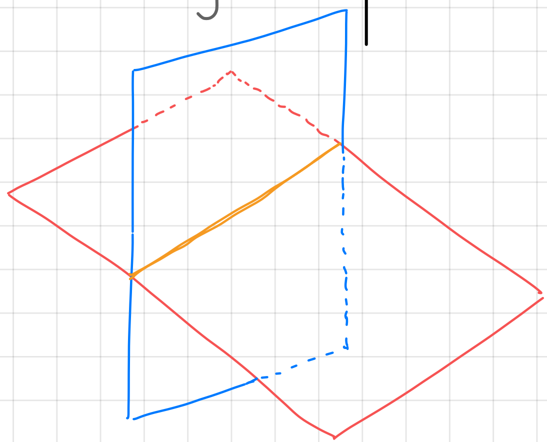
$ax + by + cz + d = 0$ general form of the equation of a plane

Parallel and intersecting planes

Let P_1 and P_2 be two planes in $\mathbb{R}^3$. Then the following possibilities exist:

		P_1 and P_2 share a common point: YES	P_1 and P_2 share a common point: NO
Normal vectors of P_1 and P_2 are parallel	YES	Equal	Parallel but not equal
	NO	Intersecting	

If two planes intersect, the intersection is a line!



Finding the line of intersection for two planes

Find the parametric and symmetric equations for the line formed by the intersection of the planes

$$x+2y+3z=0 \quad , \quad x+y+z+1=0$$

$\vec{n}_1 = \langle 1,2,3 \rangle$

$\vec{n}_2 = \langle 1,1,1 \rangle$

$k \cdot 1 = 1$

$k \cdot 1 = 2$

$k \cdot 1 = 3$

such k does not exist

$k \cdot \vec{n}_2 = \vec{n}_1$ not possible

so $\vec{n}_1$ is not parallel to $\vec{n}_2$

$$\ominus \begin{cases} x+2y+3z=0 & (1) \\ x+y+z=-1 & (2) \end{cases}$$

$y+2z=1$

$y=-2z+1$

Take $z=t$. Then $y=-2t+1$. Substitute $z=t$ and $y=-2t+1$ into (1) or (2)

$x+(-2t+1)+t=-1$

$x-2t+1+t=-1$

$x=t-2$

$$\frac{x+2}{1} = \frac{y-1}{-2} = z$$

$x+2 = t$, $\frac{y-1}{-2} = t$, $z = t$

$(t-2)+2\cdot(-2t+1)+3t$

$=t-2-4t+2+3t=0$

$$\begin{cases} x=t-2 \\ y=-2t+1 \\ z=t \end{cases}$$

parametric eq. of a line

symmetric eq. of the same line

Vector-valued functions

Definition A vector-valued function is a function that takes real numbers as inputs and gives vectors as outputs, i.e.,

$\vec{r}(t) = \langle f(t), g(t) \rangle$ - function from $\mathbb{R}$ to $\mathbb{R}^2$

$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ - fuction from $\mathbb{R}$ to $\mathbb{R}^3$

Example $\vec{r}(t) = \langle \cos t, \sin t \rangle$

$\vec{r}(t) = 2t \cdot \vec{i} - e^t \cdot \vec{j} + 0 \cdot \vec{k} = \langle 2t, -e^t, 0 \rangle$

Remark From now on we will not distinguish between the point (x, y, z) and the vector $\langle x, y, z \rangle$, both are just lists of three real numbers

Vector-valued functions

Vector valued function $\vec{r}(t)$ often represents a vector or a position in the space at time t.

Think about the motion of a planet, flight of an airplane or a bird etc.

A vector-valued function may not be defined for all real numbers. For example, $\vec{r}(t) = \langle \frac{1}{t}, \frac{1}{\cos t}, t \rangle$ is not defined for $t=0$, and $t = \frac{\pi}{2} + \pi n$, n is an integer

You can explicitly specify the set of real number for which you want to define the function by writing, e.g., $\vec{r}: [0,1] \to \mathbb{R}^3$. We call this set the domain of $\vec{r}$

Vector-valued functions

If the domain is not explicitly specified, we assume that it is the set of all real numbers for which all (three) components of $\vec{r}$ are defined

Example

$$\vec{r}(t) = \left\langle \frac{1}{t}, \frac{1}{\cos t}, t \right\rangle$$

$$\operatorname{dom}(\vec{r}(t)) = \{ t \mid t \neq 0 \text{ and } t \neq \tfrac{\pi}{2} + \pi n, \ n \text{ integer} \}$$

Sometimes the domain is found from the problem setup. If the function describes the motion of a bird between time 0 and tim T, then the domain is the interval $[0, T]$