## MATH 142A: Introduction to Analysis

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Today: Series > Q&A: February 2 Next: Ross § 15

Week 5:

• Homework 4 (due Sunday, February 6)

# Sequences Sn+1 and VISNI

Thm 12.2 Let (Sn) be a sequence, In (Sn = 0). Then

Proof. If l= D, then l ≤ B. Assume that l>0.

Take any O< l, < l. Then by Thm 9.11 (i)

Therefore,

Note that  $(\tilde{u}_{k})$  is increasing, so  $\forall k > N$ 

Now

=>

So V lic(0,1) => B is an upper bound for (0,2) =>

# Sequences Sn+1 and VISNI

#### Corollary 12.3 If lim | Sn+1 | exists, and lim | Sn+1 |= L, then n+00 | Sn

Example

Let (an) be a sequence such that VnEN an>0.

Suppose that (an) converges, lim an = a. Then

Proof. Denote Sn:= a,.... an. Then

By Corollary 12.3

#### Series

Let  $(a_n)_{n=1}^{\infty}$  be a sequence of real numbers.

For pige N, pig we denote aptaptition tag by

Def 14.1 (Infinite series) We call the expression

 $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ 

an (infinite) series. an is called the n-th term of the series. Def 14.2 (Convergent series)

We call the sum the (n-th) partial sum of the series.

If the sequence (Sn) of partial sums converges, we say that the series Zan

If lim Sn=S, then we call s the sum of the series Zan, and not write it as

### Series

- If  $\lim_{n \to \infty} s_n = +\infty (-\infty)$ , we say that  $\sum_{n=1}^{\infty} a_n$  diverges to  $+\infty (-\infty)$ and we write
- We say that Z an converges absolutely (is absolutely convergent)
  - if the series
- Remark An infinite series can be viewed as a particular
  - type of a sequence, Sn=a, +az+...+an
  - so we can use all the relevant results.
  - For example, if Vn anzo, then Sn is increasing.
  - Partial sums of Zlanl form an increasing sequence.
  - Use the criteria on convergence for partial sums etc.

#### Important examples

- is called the geometric series. 8. Let a, reR. Then If |r| < 1, then  $\sum_{n=0}^{\infty} ar^n =$ <u>Proof</u> Denote  $S_k = \sum_{n=0}^{k} ar^n =$ Note that r(1+r+-+rk)= , 50 converges iff 9. Let p>0. Then
  - $\frac{Proot(p=2).S_{k}:=\sum_{n=1}^{k}\frac{1}{n^{2}}}{(1)}(S_{k}) \text{ is increasing}$
  - (2) (Sk) is bounded

For any n=2

,50

0+2+ Thm 10.2

Cauchy criterion

Def 14.3 We say that Zan satisfies the Cauchy criterion

if its sequence of partial sums (Sn) is a Cauchy sequence, i.e.

YESO JN YM,NNN ISN-SMIKE

Thm 14.4 Zan converges <=> Zan satisfies the Cauchy criterion Proof. Follows from Thm 10.11

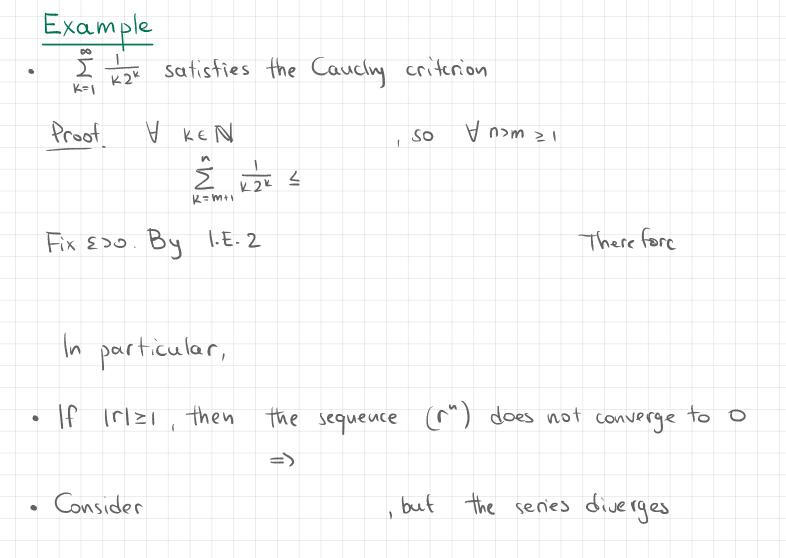
Corollary 14.5 (Necessary condition for convergence).

Zan converges =>

Proof. Zan converges =>

 $\Rightarrow$ 

<=> lim an = 0



## Comparison test

(11)

# Thm 14.6 Let (an) and (bn) be two sequence, In an 20

Then (i)

Proof. (i) Use the Cauchy criterion

Fix E>0. By Thm 14.4

By Thur 14.4 (ii) Denote Sn = Zak, tn = Zbk. Then  $\sum a_n = +\infty \iff$ 

. Then

n = 1