Math 281C Homework 6

Due: 5:00pm, May 13th

1. Find the LRT for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ based on a single observation from the density function

$$f(x) = 2\frac{\theta - x}{\theta^2}\mathbb{1}(0 < x < \theta).$$

2. Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be two independent samples with the probability density functions

$$f_1(x) = \frac{1}{\lambda_1}e^{-x/\lambda_1}\mathbb{1}(x > 0) \quad \text{and} \quad f_2(y) = \frac{1}{\lambda_2}e^{-y/\lambda_2}\mathbb{1}(y > 0),$$

respectively. We wish to test $H_0 : \lambda_1 = \lambda_2$ versus $H_1 : \lambda_1 \neq \lambda_2$.

(i). Find a UMPU test of size α.
(ii). Find an LRT of size α.
(iii). Are the two tests in (i) and (ii) the same?