Math 281C Homework 3

Due: 5:00pm, April 22nd

1. Let $X_1, \ldots, X_n$ be i.i.d. from $N(\theta, \sigma^2)$ with $\sigma^2$ known. Consider testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$. Define the test that rejects $H_0$ if and only if

$$\bar{X} > \sigma z_{\alpha/2}/\sqrt{n} + \theta_0 \quad \text{or} \quad \bar{X} < -\sigma z_{\alpha/2}/\sqrt{n} + \theta_0,$$

where $z_{\alpha/2}$ is the upper $\alpha/2$-quantile of $N(0, 1)$, and $\bar{X} = (1/n) \sum_{i=1}^n X_i$. Verify that this test a UMP unbiased (UMPU) level $\alpha$ test.

2. Let $X_1, \ldots, X_{10}$ be i.i.d. from Bernoulli($p$).
   (i) Find a UMP test of size $\alpha = 0.1$ for testing $H_0 : p \leq 0.2$ or $p \geq 0.7$ versus $H_1 : 0.2 < p < 0.7$.
   (ii) Find the power of the UMP test in (i) when $p = 0.4$.
   (iii) Find a UMP unbiased test of size $\alpha = 0.1$ for testing $H_0 : p = 0.2$ versus $H_1 : p \neq 0.2$.
   (iv) Find the power of the UMP unbiased test in (iii) when $p = 0.4$.

3. Let $X_1, \ldots, X_n$ be i.i.d. from some distribution function $F_{\theta}(x)$. Find a UMP unbiased test for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ if
   (i) $F_{\theta}(\cdot)$ is the CDF of Poisson$(\theta)$, that is,

   $$\mathbb{P}_{\theta}(X = x) = \frac{\theta^x e^{-\theta}}{x!}, \quad x = 0, 1, 2, \ldots \quad \text{and} \quad \theta > 0.$$

   In this case, $\theta_0 > 0$.
   (ii) $F_{\theta}(\cdot)$ is the CDF of Geometric$(\theta)$, that is,

   $$\mathbb{P}_{\theta}(X = x) = (1 - \theta)^{x-1} \theta, \quad x = 1, 2, \ldots \quad \text{and} \quad 0 \leq \theta \leq 1.$$

   In this case, $0 < \theta_0 < 1$.

4. Let $X$ and $Y$ be independently distributed with Poisson distributions Poisson($\lambda$) and Poisson($\mu$). Find the power of the UMP unbiased test of $H_0 : \mu \leq \lambda$ versus the alternative $\lambda = 1, \mu = 2$. at level of significance $\alpha = 0.1$. 