Math 281C Homework 3

Due: 5:00pm, April 22nd

1. Let $X_1, \ldots, X_n$ be i.i.d. from $N(\theta, \sigma^2)$ with σ^2 known. Consider testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$. Define the test that rejects H_0 if and only if

$$\bar{X} > \sigma z_{\alpha/2}/\sqrt{n} + \theta_0 \quad \text{or} \quad \bar{X} < -\sigma z_{\alpha/2}/\sqrt{n} + \theta_0,$$

where $z_{\alpha/2}$ is the upper $\alpha/2$-quantile of $N(0,1)$, and $\bar{X} = (1/n)\sum_{i=1}^n X_i$. Verify that this test a UMP unbiased (UMPU) level α test.

2. Let $X_1, \ldots, X_{10}$ be i.i.d. from Bernoulli(p).
   (i) Find a UMP test of size $\alpha = 0.1$ for testing $H_0 : p \leq 0.2$ or $p \geq 0.7$ versus $H_1 : 0.2 < p < 0.7$.
   (ii) Find the power of the UMP test in (i) when $p = 0.4$.
   (iii) Find a UMP unbiased test of size $\alpha = 0.1$ for testing $H_0 : p = 0.2$ versus $H_1 : p \neq 0.2$.
   (iv) Find the power of the UMP unbiased test in (iii) when $p = 0.4$.

3. Let $X_1, \ldots, X_n$ be i.i.d. from some distribution function $F_\theta(x)$. Find a UMP unbiased test for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ if
   (i) $F_\theta(\cdot)$ is the CDF of Poisson(θ), that is,

$$\mathbb{P}_\theta(X = x) = \theta^x e^{-\theta}/x!, \quad x = 0, 1, 2, \ldots \quad \text{and} \quad \theta > 0.$$

   In this case, $\theta_0 > 0$.
   (ii) $F_\theta(\cdot)$ is the CDF of Geometric(θ), that is,

$$\mathbb{P}_\theta(X = x) = (1-\theta)^{x-1}\theta, \quad x = 1, 2, \ldots \quad \text{and} \quad 0 \leq \theta \leq 1.$$

   In this case, $0 < \theta_0 < 1$.

4. Let X and Y be independently distributed with Poisson distributions Poisson(λ) and Poisson(μ). Find the power of the UMP unbiased test of $H_0 : \mu \leq \lambda$ versus the alternative $\lambda = 1, \mu = 2$. at level of significance $\alpha = 0.1$.