Math 281C Homework 1

Due: 5:00pm, April 8th

1. The random variable X has p.d.f $f(x) = e^{-x}$, $x > 0$. One observation is obtained on the random variable $Y = X^{\theta}$, and a test of $H_0 : \theta = 1$ versus $H_1 : \theta = 2$ needs to be constructed. Find the UMP level $\alpha = 0.1$ test, and compute the Type II error probability.

2. Let X_1, X_2 be iid Uniform$(\theta, \theta + 1)$. For testing $H_0 : \theta = 0$ versus $H_1 : \theta > 0$, we have two competing tests

$$\phi_1(X_1) : \text{Reject } H_0 \text{ if } X_1 > 0.95,$$
$$\phi_2(X_1, X_2) : \text{Reject } H_0 \text{ if } X_1 + X_2 > C.$$

   (a) Find the value C so that ϕ_2 has the same size as ϕ_1.
   (b) Calculate the power function of each test. Draw a well-labeled graph of each power function.
   (c) Prove or disprove: ϕ_2 is a more powerful test that ϕ_1.
   (d) Show how to get a test that has the same size but is more powerful than ϕ_2.

3. Show that for a random sample $X_1, \ldots, X_n$ from a $N(0, \sigma^2)$ population, the most powerful test of $H_0 : \sigma = \sigma_0$ versus $H_1 : \sigma = \sigma_1$, where $\sigma_0 < \sigma_1$, is given by

$$\phi(T) = \begin{cases} 1 & \text{if } T > c \\ 0 & \text{if } T \le c, \end{cases}$$

   where $T = T(X_1, \ldots, X_n) = \sum_{i=1}^{n} X_i^2$. For a given value of α (the size of the Type I error), show how the value of c is explicitly determined.