Math 281C Homework 4

Due: May 8, 5 pm

1. A random variable X with mean μ is called *sub-Gaussian* if there is a positive parameter σ such that

$$\mathbb{E}e^{\lambda(X-\mu)} \le e^{\sigma^2\lambda^2/2}$$

for any $\lambda \in \mathbb{R}$. σ is referred to as the sub-Gaussian parameter. Suppose $\{X_i\}_{i=1}^n$ are independent with means $\{\mu_i\}_{i=1}^n$ and sub-Gaussian parameters $\{\sigma_i\}_{i=1}^n$. Show that for any $t \ge 0$,

$$\mathbb{P}\left(\sum_{i=1}^n (X_i - \mu_i) \ge t\right) \le \exp\left(-\frac{t^2}{2\sum_{i=1}^n \sigma_i^2}\right).$$

2. Define the class of matrices

$$\mathbb{M}_{(1)}^{n,d} := \{\Theta \in \mathbb{R}^{n\times d} : \mathrm{rank}(\Theta) = 1, \|\Theta\|_{\mathrm{F}} = 1\},$$

where $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm. Prove that for any $0 < \epsilon < 1$,

$$\log N(\epsilon, \mathbb{M}_{(1)}^{n,d}, \|\cdot\|_{\mathrm{F}}) \le (n+d)\log(1 + 4/\epsilon).$$