Math 281C Homework 2

Due: Apr 24, 5 pm

1. Suppose that $X \geq 0$ and that the moment generating function of X exists in an interval around zero. Given some $\delta > 0$ and integer $k = 0, 1, 2, \ldots$, show that
$$\inf_{k=0,1,2,\ldots} \frac{\mathbb{E}[|X|^k]}{\delta^k} \leq \inf_{\lambda>0} \frac{\mathbb{E}[e^{\lambda X}]}{e^{\lambda\delta}}.$$

2. Let $\{X_i\}_{i=1}^n$ be i.i.d. random variables drawn from a density f on the real line. Recall from Math 281A that a standard estimate of f is the kernel density estimate
$$\widehat{f}_n(x) := \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - X_i}{h}\right),$$
where $K : \mathbb{R} \to [0, \infty)$ is a kernel function satisfying $\int_{-\infty}^{\infty} K(t)\mathrm{d}t = 1$, and $h > 0$ is a bandwidth parameter. Suppose that we assess the quality of $\widehat{f}_n$ using the ℓ_1-norm
$$\|\widehat{f}_n - f\|_1 := \int_{-\infty}^{\infty} |\widehat{f}_n(t) - f(t)|\mathrm{d}t.$$
Prove that
$$\mathbb{P}(\|\widehat{f}_n - f\|_1 \geq \mathbb{E}[\|\widehat{f}_n - f\|_1] + \delta) \leq e^{-n\delta^2/2}.$$