Math 281A Homework 7

Due: Dec 5, in class

1. Let $\{X_i\}_{i=1}^n$ be i.i.d. from Pareto distribution with density

$$f(x) = \frac{\theta c^\theta}{x^{\theta+1}} I\{x \geq c\},$$

for known constant $c > 0$ and parameter $\theta > 0$. Derive the Wald, Rao, and likelihood ratio tests of $\theta = \theta_0$ against a two-sided alternative.

2. Suppose $\mathbf{X} \sim \text{multinomial}(n, \mathbf{p})$, where $\mathbf{p} \in \mathbb{R}^k$, and define $\boldsymbol{\theta} = (p_1, \dots, p_{k-1})$. Suppose we wish to test $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0$ against $H_1 : \boldsymbol{\theta} \neq \boldsymbol{\theta}_0$.
   (a) Prove that the Wald and score tests are the same as the usual Pearson χ^2 test;
   (b) Derive the likelihood ratio statistic.

3. Show that under sufficient regularity conditions, the mean integrated square error (MISE) of a kernel estimator $\hat{f}_n$ with a bandwidth parameter h satisfies

$$\text{MISE}_f(\hat{f}_n) \sim \frac{1}{nh} \int K^2(y) \mathrm{d}y + \frac{h^4}{4} \int f''(x)^2 \mathrm{d}x \left(\int y^2 K(y) \mathrm{d}y \right)^2.$$

What does this imply for an optimal choice of the bandwidth h?