

Math 281A Homework 6

Due: Nov 21, in class

1. Let $\{X_i, Y_i\}_{i=1}^n$ be i.i.d. random vectors with $Y_i \in \{0, 1\}$, and

$$\mathbb{P}_{\alpha, \beta}(Y_i = 1 | X_i = x) = \frac{1}{1 + e^{-\alpha - \beta x}}.$$

The distribution of X_i is non-degenerate, but unknown. Do we have closed form of MLE $(\hat{\alpha}, \hat{\beta})$? Derive the asymptotic distribution of $(\hat{\alpha}, \hat{\beta})$.

2. Let $\{X_i\}_{i=1}^n$ be i.i.d. from $\text{Poisson}(1/\theta)$.
- (a) Calculate the Fisher information I_θ in one observation;
 - (b) Derive the MLE $\hat{\theta}$ and show its asymptotic distribution.
3. Let $\{X_i\}_{i=1}^n$ be i.i.d. from $N(\theta, \theta)$.
- (a) Calculate the Fisher information I_θ in one observation;
 - (b) Derive the MLE $\hat{\theta}$ and show its asymptotic distribution.
4. (a) Calculate the Kullback-Leibler divergence between two exponential distributions with different scale parameters, when is it maximal?
- (b) Calculate the Kullback-Leibler divergence between two normal distributions with different location and scale parameters, when is it maximal?