Math 281A Homework 5

Due: Nov 14, in class

1. Let $\{x_i\}_{i=1}^n$ be i.i.d. sample from a strictly positive density that is symmetric about θ, show that the Huber M-estimator for location is consistent for θ.

2. Let $\{x_i\}_{i=1}^n$ be i.i.d. sample from a strictly positive density. Define

$$\psi(x) = \frac{2}{1+e^{-x}} - 1,$$

and $\hat{\theta}_n$ be the solution of

$$\sum_{i=1}^{n} \psi(X_i - \theta) = 0.$$

   (a) Show that $\hat{\theta}_n \xrightarrow{P} \theta_0$ for some θ_0, and express θ_0 in the density of observations;
   (b) Show that $\sqrt{n}(\hat{\theta}_n - \theta_0)$ converges in distribution and find the limit variance.

3. Let $\{x_i\}_{i=1}^n$ be i.i.d. sample from Uniform$(0,1)$, determine the relative efficiency of the sample median and the sample mean.

4. Let $\{x_i\}_{i=1}^n$ be i.i.d. sample from $N(\theta,1)$, find the relative efficiency of the Huber estimator and the sample mean.