

Math 281A Homework 4

Due: Oct 31, in class

1. Let $\{X_i\}_{i=1}^n$ be an i.i.d. sample from Poisson distribution with mean θ . Find a variance stabilizing transformation for the sample mean and construct a confidence interval for θ based on this.
2. Let $X_1 \sim \text{Uniform}(0, 2\pi)$, and let $X_2 \sim \exp(1)$, independent of X_1 . Find the joint distribution of $(Y_1, Y_2) = (\sqrt{2X_2} \cos X_1, \sqrt{2X_2} \sin X_1)$.
3. Let $\{X_i\}_{i=1}^n$ be i.i.d. from logistic distribution with cdf

$$F_\theta(x) = \frac{e^{t/\theta}}{1 + e^{t/\theta}}, \quad \text{for } t \in \mathbb{R}.$$

- (a) Find the asymptotic distribution of $X_{(n)} - X_{(n-1)}$;
- (b) Based on part (a), construct a 95% confidence interval for θ . You can use the fact that 0.025 and 0.975 quantiles of standard exponential distribution are 0.0253 and 3.6889;
- (c) Simulate 1000 samples of size $n = 40$ and $\theta = 2$. How many confidence intervals contain θ ?