Math 281A Homework 3

Due: Oct 24, in class

1. The Pareto distribution possesses the density function

$$f_{\alpha,\mu}(x) = \frac{\alpha\mu^\alpha}{x^{\alpha+1}} I\{x \geq \mu\}$$

with $\alpha, \mu > 0$. Denote $\hat{\alpha}_n$ as the MLE of a based on an i.i.d. sample $\{X_i\}_{i=1}^n$. Determine the limit distribution of $\sqrt{n}(\hat{\alpha}_n - \alpha)$ when

   (a) μ is known;

   (b) μ is unknown.

2. Let $\{X_i\}_{i=1}^n$ be an i.i.d. sample from Poisson distribution with mean θ. Find a variance stabilizing transformation for the sample mean and construct a confidence interval for θ based on this.

3. Suppose $\phi : \mathbb{R} \to \mathbb{R}$ is twice continuously differentiable at θ with $\phi'(\theta) = 0$ and $\phi''(\theta) \neq 0$. Suppose that $\sqrt{n}(T_n - \theta) \xrightarrow{d} N(0, 1)$.

   (a) Show that $\sqrt{n}(\phi(T_n) - \phi(\theta)) \xrightarrow{P} 0$;

   (b) What is the asymptotic distribution of $n(\phi(T_n) - \phi(\theta))$? Justify your answer.

4. Let $X_1, \ldots, X_n$ be i.i.d. from density $f_{\lambda,a}(x) = \lambda e^{-\lambda(x-a)} I(x \geq a)$, where $\lambda > 0$ and $a \in \mathbb{R}$ are unknown parameters. Recall that we found the MLE $(\hat{\lambda}_n, \hat{a}_n)$ of (λ, a) in homework 1, derive the asymptotic property of $(\hat{\lambda}_n, \hat{a}_n)$.