Hiding the constant drift and the perturbed Tanaka equation

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Suppose we have a continuous process in some filtration. Even if it is not a Brownian motion (BM) in the given filtration in some cases it is a BM in its own filtration. Assume now, that we have S a Brownian motion with constant drift in some filtration and we can take an integral $\beta = H \cdot S$ with respect to this process S. Is it possible to choose the integrand H in such a way that the result is a BM in its own filtration. The idea of the solution is to take an additional uniform random variable U independent of S and define the integrand such that it takes plus or minus one according to U is smaller or bigger than its conditional median given $\mathcal{F}_t^\beta$. It is possible to derive a solution in this way, however the integrand obtained from this median rule will not be adapted to the filtration of S. It turns out that the existence of a strong solution, that is, H adapted to the filtration of S, is related to the strong solvability of an SDE. This later problem is also related to the lack of the semimartingale property of reflected BM in the orthant and also to some classical local time method of proving strong uniqueness of SDE. Finally, it raises the question that in what generality is it true that a strong enough additive noise restores the strength of the solution of an SDE.