Finite dimensional approximations of hyperfinite martingales

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Inspired by the classical theory, non-commutative probability is motivated by quantum mechanics. Suitable algebras of operators on a Hilbert space (von Neumann algebras) are the natural framework for non-commutative integration and martingale theory.

The non-commutative analogues of the Burkholder-Gundy square function inequalities were established by Pisier and Xu. They also proved that, under certain conditions on the filtration, a non-commutative martingale can be transferred to a commutative vector-valued martingale, where classical theory applies. We will show that in the setting of a hyperfinite von Neumann algebra, an L_p-martingale can be approximated in the p norm ($1 < p < \infty$) by martingales with respect to finite dimensional filtrations, for which Pisier-Xu's argument applies. The class of hyperfinite von Neumann algebras includes the algebra of bounded linear operators on a separable Hilbert space (in particular, matrix algebras), the classical L_∞-spaces, the Clifford algebra, group von Neumann algebras associated to amenable groups and is closed under tensor products.