Lévy's continuity theorem for general convolution structures

The crucial part of the continuity theorem of classical probability theory asserts that given a sequence $(\mu_n)_{n\geq 1}$ of bounded (nonnegative) measures on $\mathbf{R^d}$ and a function h on $\mathbf{R^d}$ which is continuous at o such that the sequence $(\hat{\mu}_n)_{n\geq 1}$ of Fourier transforms converges pointwise on $\mathbf{R^d}$ to h, there exists a bounded measure μ on $\mathbf{R^d}$ such that $(\mu_n)_{n\geq 1}$ converges weakly to μ.

Any attempt of extending this statement beyond the Euclidean space consists in reformulating it as the Lévy continuity property (LCP) and in showing that this property holds in more general situations. Much work has been done since Lévy's initiative of 1924 in two directions: to replace $\mathbf{R^d}$ by some algebraic-topological structure K like a group or a hypergroup and the Fourier transform $\mu \mapsto \hat{\mu}$ by an appropriate integral transform on the space of all bounded measures on K.

We shall discuss (LCP) in the case of completely regular abelian groups and deal in detail with the case of not necessarily commutative locally compact groups K. In the latter case new results can be achieved by applying the Bishop-deLeeuw integral representation to suitable convex sets of positive definite functions on K.