Efficient Nonparametric Estimation of Spectral Functionals for Continuous-time Gaussian Stationary Models

Mamikon S. Ginovyan (Boston University)
e-mail: ginovyan@math.bu.edu

Abstract

The problem of asymptotically efficient estimation of different kind of functionals for various statistical models has been extensively discussed in the literature. This talk is concerned with the estimation of spectral functionals for continuous-time Gaussian stationary models.

Suppose we observe a finite realization $\{X(t),\, 0 \leq t \leq T\}$ of a centered real-valued stationary Gaussian process $\{X(t),\, t \in \mathbb{R}\}$ with an *unknown* spectral density $\theta(\lambda)$. Assume that $\theta(\lambda)$ belongs to a given (infinite-dimensional) class $\mathbf{\Theta}$ of spectral densities possessing some smoothness properties. Let $\Phi(\cdot)$ be some *known* functional, the domain of definition of which contains $\mathbf{\Theta}$. The problem is to estimate the value $\Phi(\theta)$ of the functional $\Phi(\cdot)$ at an unknown point $\theta \in \mathbf{\Theta}$, and investigate the asymptotic properties of the suggested estimators, depending on the dependence structure of the model and smoothness structure of the "parametric" set $\mathbf{\Theta}$. The main objective is construction of asymptotically efficient estimators for $\Phi(\theta)$.

We define the concepts of H- and IK-efficiency of estimators, based on the variants of Hájek convolution theorem and Hájek-Le Cam local asymptotic minimax theorem, respectively, and show that the simple "plug-in" statistic $\Phi(I_T)$, where $I_T = I_T(\lambda)$ is the periodogram of the underlying process $X(t)$, is H- and IK-asymptotically efficient estimator for a linear functional $\Phi(\theta)$, while for a nonlinear smooth functional $\Phi(\theta)$, an H- and IK-asymptotically efficient estimator is the statistic $\Phi(\widehat{\theta}_T)$, where $\widehat{\theta}_T$ is a suitable sequence of the so-called "undersmoothed" kernel estimators of the unknown spectral density $\theta(\lambda)$.

Exact asymptotic bounds for minimax mean square risks of estimators of linear functionals will also be presented.