

4.9 Exercises

For the following exercises, you should assume that the stock follows a Black-Scholes model and that the current value of a contingent claim is the current arbitrage free price of the contingent claim.

1. Verify that in the Example in Section 4.1, ϕ is not an admissible strategy.
2. Suppose that time is measured in years and that the riskless interest rate is 5% per year (compounded continuously). Assume that the stock moves according to an exponential Brownian motion with drift parameter $\mu = 0.15$, volatility parameter $\sigma = 0.20$ and has a current price of \$10.
 - (a) Consider a European call option on the stock with an exercise price of \$12 and an expiration date of six months from now. What is the value of this option now? Describe a replicating strategy for this option.
 - (b) What is the current value of a European contingent claim that pays \$1 if the stock price is more than \$10 in six month's time and pays off zero otherwise.
 - (c) By numerically approximating solutions of stochastic differential equations, or otherwise, estimate the current value of a European contingent claim called an *Asian* option that has a value at its expiration six months from now of

$$\left(2 \int_0^{0.5} S_t dt - K \right)^+ \quad (4.90)$$

where S_t denotes the stock price t years from now and $K = \$12$.

3. Consider a European call option with a strike price of \$70 and an expiration date of one month from now. The option is based on a stock with a current price of \$82. Suppose that the current price of the option is \$14. Assuming that the risk free interest rate is 6% per year, estimate the implied volatility of the stock. Please indicate the units in which your answer is measured. (You may use Newton's method or some other suitable numerical scheme. You will also need to use a reasonable approximation for the number of trading days between now and one month's time. Please use 21 days or 1/12 of a year for this.) You may wish to check your answer against the option pricing "black boxes" available on the web. For example, from the class web page you can go to the Courant Finance server and check that the implied volatility you found is consistent with the given option price and other data. Alternatively, you can use the implied volatility calculator available at the Dailystocks.com web site.

4.

- (a) This part of the exercise uses the listing from the Wall Street Journal given out on the first day of class. Consider the call options on CISCO stock

that were to expire in January 2001. These all expired on the same day in January, but had differing strike prices and differing initial prices. For each of these options, assuming a Black-Scholes model and that these are European options, estimate the implied volatility. (Assume that the 3-month T-bill rate is the risk free interest rate.) Make a graph of implied volatility versus strike price from these calculations.

- (b) This part of the exercise requires you to obtain historical data on the daily closing price of CISCO stock for the same period that the options in part (a) were active. Such data can be readily obtained from the Yahoo finance web site (a link to this is available from the Math 294 web site). Assuming the Black-Scholes model, consider the approximation for the stock price S over the interval $[0, T]$ given by

$$S_{t+\Delta t} - S_t = \mu S_t \Delta t + \sigma S_t \Delta_t W, \quad (4.91)$$

where $\Delta t = T/n$ (some positive integer n), t is a multiple of Δt , $\Delta_t W = W_{t+\Delta t} - W_t$ is an increment of the Brownian motion W and it is normally distributed with mean 0 and variance Δt . Here the natural choice for Δt , given daily closing price data, is one day. (In our time line, we ignore non-trading days such as weekends and holidays.) For $i = 1, \dots, n \equiv \frac{T}{\Delta t}$, let

$$X_i = \frac{S_{i\Delta t} - S_{(i-1)\Delta t}}{S_{(i-1)\Delta t}}. \quad (4.92)$$

Then, the usual estimators for μ and σ^2 are given by

$$\bar{\mu} = \frac{1}{T} \sum_{i=1}^n X_i, \quad \bar{\sigma}^2 = \frac{1}{(n-1)\Delta t} \sum_{i=1}^n (X_i - \bar{\mu}\Delta t)^2. \quad (4.93)$$

Apply the above to the closing stock price data for CISCO stock over the period covered by the options considered in part (a). Compare the estimate $\bar{\sigma}$ of the volatility obtained in this way with the estimates obtained from the option prices.

- 5.** Prove that the process $\{U_t, t \in [0, T]\}$ obtained in the discussion of American contingent claims is the arbitrage free price process for an American contingent claim. (For this, you will need to develop a notion of arbitrage for trading in stock, bond and the American contingent claim. You should make sure to indicate this definition, cf. Section 4.4.)