Math 194
Introduction to the
Mathematics of Finance
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Chapter 1

Financial Markets and Derivatives

1.1 Financial Markets

A *financial market* consists of tradable securities such as stocks, bonds, currencies, commodities, or even indexes (see the excerpt from the Wall Street Journal). One reason for the existence of financial markets is that they facilitate the flow of capital. For example, if a company wants to finance the building of a new production facility, it might sell shares of stock to investors who buy the shares based on the anticipation of future rewards, such as dividends or a rise in the stock price.

A variety of stochastic models are used in modeling the prices of securities. All such models face the usual trade off, namely, more complex models typically provide a better fit to data, whereas simpler models are generally more tractable and despite their simplicity can sometimes provide useful qualitative insights. Finding a good balance between a realistic and a tractable model is part of the art of stochastic modeling. For reasons of tractability (to avoid measure theory), mostly discrete models will be considered here.

An *arbitrage* opportunity is an opportunity for a risk free profit. A financial market is said to be *viable* if there are no arbitrage opportunities. Typically, liquid financial markets move rapidly to eliminate arbitrage opportunities.

1.2   Derivatives

A *derivative* or *contingent claim* is a security whose value depends on the value of some underlying security. Examples of derivatives are forward contracts (traded over the counter, i.e., traded between individual buyers and sellers), futures contracts (essentially forward contracts traded on a financial exchange), options, swaps, etc. The underlying security on which a derivative is based could be a security in a financial market, such as a stock, bond, currency or commodity, but it could also be a derivative itself, such as a futures contract. Secondary financial markets can be formed from derivatives.

A *forward contract* is an agreement to buy or sell an asset at a certain future time for a certain price.

An *option* is a contract which gives the holder of the option the right, but not the obligation, to buy or sell a given security at a given price (called the exercise or *strike price)* within a fixed time period $[0, T]$. A *call option* gives the option holder the right to buy at the given price, whereas a *put option* gives the option holder the right to sell at the given price. A *European option* can only be exercised by the holder of the option at the expiration time T, whereas an *American option* can be exercised by the holder at any time in $[0, T]$.

Remark. Options based on stocks are usually written to cover the buying or selling of 100 shares of a stock. This is convenient since shares of stock are usually traded in lots of 100. Most exchange traded options on stock are American-style options. Stock options on the Chicago Board Options Exchange expire on Saturday immediately following the third Friday of the expiration month.

The "derivatives market" is huge – a book written in 1996 (by Baxter and Rennie) described it then as a $15 trillion market, and a recent book by S. Pliska cites it as having $20 trillion in principal outstanding. One might ask, why are options worth buying or selling? To help answer this, consider an example. To keep the description simple, consider a European call option.

Example 1 On January 4, 2000, a European call option on CISCO stock has a price of $33. The option expires in January and the strike price is $70. The price of CISCO stock on January 4 is $102.

If one bought such an option on 100 shares of CISCO, the option would cost $3,300, and on January 21, 2000 (third Friday of January), one would have the right to buy 100 shares of CISCO at a price of $70 per share. Suppose for simplicity that $1 on January 4 is worth $1 on January 21, 2000.

Scenario 1: Suppose the price of CISCO stock on January 21 is $120 per share. The holder of the option will exercise it and make a net profit per share of $120 -$70 -$33 (spot price of stock on January 21 - price under exercise of option - option price) and hence a net profit of $1700. This is a $1700/33\% = 51.5\%$ profit on the $3,300 investment. On the other hand, if the $3,300 had been directly

invested in stock, the profit would have been $\$18 \times 32 = 576$ (on an investment of $\$ 102 \times 32 = \$3,264$ which is a $57600/3264\% = 17.6\%$ profit.

Scenario 2: Suppose the price of CISCO stock on January 21 is $\$67$ per share. The holder of the option will not exercise it and makes a loss of $\$33$ per share (the cost of the option per share) and hence a net loss of $\$3,300$. This is a 100% loss on the $\$3,300$ initial investment. On the other hand, if the $\$3,300$ had been invested directly in stock, the loss would have been $35 \times 32 = \$1120$ or a 34.3% loss on an investment of $\$3,264$ in stock.

There are two main uses of derivatives, namely, *speculation and hedging.* For example, the buyer of a call option on a stock is leveraging his/her investment. The option may not cost much compared to the underlying stock and the owner of the call option can benefit from a rise in the stock price without having to buy the stock. Of course, if the stock price goes down substantially and the option is not exercised, the buyer of the call loses what he/she paid for the call option. In this case, the seller of the call option would benefit, without necessarily having to buy the stock. Of course, if the stock price goes up substantially and the option seller does not already own the stock, then he/she will have to pay the price of buying the stock at the increased price and turning it over to the owner of the option. Options may be used to leverage and speculate in the market. Depending on how the investment is used, the down side losses can be substantial.

On the other hand, options can be used to hedge risk. For example, an owner of stock who wants to hedge against a dramatic decrease in the stock price could buy a put option to sell the stock at a certain price.

1.3 Exercises

1. Using the list of option prices given out in class, perform a similar calculation to that done in Example 1 of the text with AmOnline (America Online) in place of CISCO. For this, use the first AmOnline call option expiring in January for which a price is listed. You should assume that a European call option for 100 shares of stock is purchased and that at expiration there are two possible scenarios for the stock price – it has gone up or down by 10% since purchase of the option.