Homework #8 **Due March 13** **Math 194**

1. Let $\Omega = \{\omega_1, \omega_2\}$ with a subjective probability $P : P(\omega_i) > 0$, $i = 1, 2$.

   Consider a single period model with one riskless and one risky asset. Suppose the riskless asset $S^0 = \{S_0^0, S_1^0\}$ satisfies $S_t^0 = (1+r)^t$, $t = 0, 1$ for $r = \frac{1}{9}$, and suppose the risky asset $S^1 = \{S_0^1, S_1^1\}$ is such that $S_0^1 = 5$, $S_1^1(\omega_1) = \frac{20}{3}$, $S_1^1(\omega_2) = \frac{40}{9}$. The discounted value of S_1^1 is given by $S_1^{*,1} = S_1^1 / S_1^0$ and $\Delta S_1^{*,1} = S_1^{*,1} - S_0^1$.

   Consider the optimal portfolio selection problem

$$(OP) \begin{cases} \text{maximize } \{E^P[u(V_1)] : \ H \in \mathcal{H}\} \\ \text{subject to } V_0 = \nu \end{cases}$$

   where $\mathcal{H}$ is the set of self-financing trading strategies $H = (H^0, H^1)$ and

$$V_1 = V_1(H) = H^0 S_1^0 + H^1 S_1^1 = (V_0(H) + H^1 \Delta S_1^{*,1}) S_1^0$$

   is the value of H at time one and $V_0 = V_0(H)$ is the initial value of H. Here, the initial wealth ν is assumed to satisfy $\nu \geq 0$.

   For each of the following utility functions, use the risk neutral computational approach to find the solution of (OP).

   (a) $u(x) = \ln x$

   (b) $u(x) = -\exp(-x)$

   (c) $u(x) = \gamma^{-1} x^\gamma$ where $-\infty < \gamma < 1$, $\gamma \neq 0$.