HOMEWORK 7 **Due March 6, 2001**

1. Let $T = 2$, $\Omega = \{\omega_1, \dots, \omega_4\}$, $P(\{\omega_i\}) > 0$ for $i = 1, \dots, 4$, $\mathcal{F}_0 = \{\emptyset, \Omega\}$, $\mathcal{F}_1 = \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \emptyset, \Omega\}$, and $\mathcal{F}_2$ be the collection of all subsets of Ω. Consider a riskless asset with price process $S^0 = \{S^0_t, t = 0, 1, 2\}$ where $S^0_t = 1$ for all t, and a risky asset with price process $S^1 = \{S^1_t, t = 0, 1, 2\}$ such that

$$S^1_0(\omega_1) = 5, \quad S^1_1(\omega_1) = 8, \quad S^1_2(\omega_1) = 9 \tag{1}$$
$$S^1_0(\omega_2) = 5, \quad S^1_1(\omega_2) = 8, \quad S^1_2(\omega_2) = 6 \tag{2}$$
$$S^1_0(\omega_3) = 5, \quad S^1_1(\omega_3) = 4, \quad S^1_2(\omega_3) = 6 \tag{3}$$
$$S^1_0(\omega_4) = 5, \quad S^1_1(\omega_4) = 4, \quad S^1_2(\omega_4) = 3. \tag{4}$$

Then

$$X = \max(0, S^1_0 - 7, S^1_1 - 7, S^1_2 - 7), \tag{5}$$

is the value at time T of a so-called *look-back* option, where this value depends on the prices of the underlying asset S^1 in the past as well as at time T.

(a) Draw a tree to indicate the possible "paths" followed by the risky asset price process S^1.

(b) Find an equivalent martingale measure for the model.

(c) Find a replicating strategy for the option whose value at time T is given by X.

(d) What is the arbitrage free price for the option at time zero?

2. Let $T = 1$, $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $r = \frac{1}{9}$. There are two risky assets with processes S^1 and S^2:

		S^i_1			
i	S^i_0	ω_1	ω_2	ω_3	ω_4
1	5	20/3	20/3	40/9	20/9
2	10	40/3	80/9	80/9	120/9

(a) Characterize the set of all replicable European contingent claims X.

(b) Compute $V_+(X)$ and $V_-(X)$ for $X = (40, 30, 20, 10)$, i.e., $X(\omega_1) = 40$, $X(\omega_2) = 30$, $X(\omega_3) = 20$, $X(\omega_4) = 10$.