Winter 2001

Homework #6 — Math 194

Exercise 1:

Take $T = 1$, $r = \frac{1}{9}$.
$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$.
There are two risky assets with price processes S^1, S^2:

		S_1^i			
i	S_0^i	ω_1	ω_2	ω_3	ω_4
1	5	20/3	20/3	40/9	20/9
2	10	40/3	80/9	80/9	40/3

(i) Specify the space

$$L = \{G_T^*(H) : H \text{ is a self-financing trading strategy with } V_0(H) = 0\}$$

for this example.

(ii) Are there any abitrage opportunities for this example? If so, find all of them.

(iii) Are there any risk neutral probabilities for this example? If so, find all of them.

Exercise 2:

Take $T = 2$, $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$, $r = 0$. There is one risky asset with price process $S^1 = \{S_0^1, S_1^1, S_2^1\}$.

$$\begin{array}{lll}
S_0^1(\omega_1) = 6 & S_1^1(\omega_1) = 5 & S_2^1(\omega_1) = 3 \\
S_0^1(\omega_2) = 6 & S_1^1(\omega_2) = 5 & S_2^1(\omega_2) = 4 \\
S_0^1(\omega_3) = 6 & S_1^1(\omega_3) = 5 & S_2^1(\omega_3) = 8 \\
S_0^1(\omega_4) = 6 & S_1^1(\omega_4) = 7 & S_2^1(\omega_4) = 6 \\
S_0^1(\omega_5) = 6 & S_1^1(\omega_5) = 7 & S_2^1(\omega_5) = 8
\end{array}$$

(i) Draw a tree to represent the possible paths following by S^1.

(ii) Find the set of all risk neutral probabilities.

Exercise 3 (p. 16, Ex. 1.10 of Pliska's book)

Consider $T = 1$, $\Omega = \{\omega_1, \ldots, \omega_n\}$. Let A denote the $(n+1) \times (n+2d)$ matrix

$$\begin{bmatrix} 0 & \ldots & \ldots & 0 & 1 & 1 & \ldots & 1 \\ \Delta S_1^{*,1}(\omega_1) & -\Delta S_1^{*,1}(\omega_1) & \ldots & -\Delta S_1^{*,d}(\omega_1) & -1 & 0 & \ldots & 0 \\ \Delta S_1^{*,1}(\omega_2) & -\Delta S_1^{*,1}(\omega_2) & \ldots & -\Delta S_1^{*,d}(\omega_2) & 0 & -1 & \ldots & 0 \\ \vdots & & & & \vdots & & & \\ \Delta S_1^{*,1}(\omega_n) & -\Delta S_1^{*,1}(\omega_n) & \ldots & -\Delta S_1^{*,d}(\omega_n) & 0 & 0 & \ldots & -1 \end{bmatrix}$$

Let b denote the $(n+1)$-vector $(1, 0, \ldots, 0)'$. Show that

$$Ax = b, \quad x \geq 0, \quad x \in \mathbb{R}^{n+2d}$$

has a solution if and only if there exists an arbitrage opportunity.