Math 194: Homework 5, due Tuesday February 13, 2001.

For the following exercise, you should assume that the stock price follows a (continuous) Black-Scholes model so that under the risk neutral probability P^*, for any $t \in [0, T]$,

$$S_t = S_0 \exp\left(\left(r - \frac{1}{2}\sigma^2\right) t + \sigma U_t\right),$$

where U_t has a normal distribution with mean 0 and variance t. Then if X represents the value at time T for a European contingent claim and X (or equivalently X^*) has finite mean under P^*, then the arbitrage free initial price for the contingent claim is given by

$$E^{P^*}[X^*]$$

where $X^* = Xe^{-rT}$.

In class it was shown that if X corresponds to a European call option with strike price K, i.e., $X = (S_T - K)^+$, then

$$E^{P^*}[X^*] = S_0 \Phi(d_+) - e^{-rT} K \Phi(d_-)$$

where Φ is the cumulative standard normal distribution function and

$$d_+ = \frac{\log(S_0/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_- = \frac{\log(S_0/K) + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

1. Suppose that time is measured in years and that the riskless interest rate is 5% per year (compounded continuously). Assume that the volatility parameter $\sigma = 0.20$ and the initial stock price is $10.

(a) Consider a European call option on the stock with an exercise price of $12 and an expiration date of six months from the initial time. Compute the initial arbitrage free price for this option.

(b) Consider a European contingent claim that expires six months after the initial time and which at expiry is worth $1 if the stock price at that time is more than $10 and is worth zero otherwise. Write down a formula for the value of the contingent claim at the expiration time in terms of the stock price. Compute the initial arbitrage free price for this contingent claim.