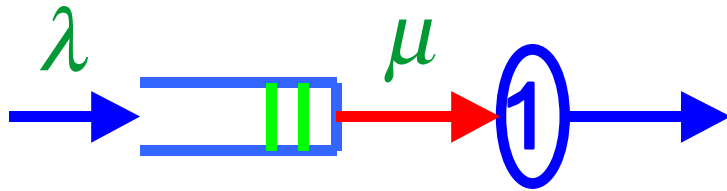


LECTURE 2

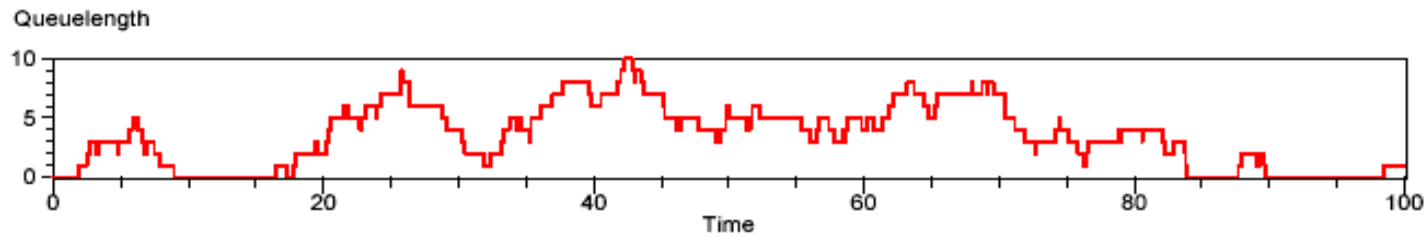
CRITICAL LOADING (HEAVY TRAFFIC)

M/M/1 Queue

(Simulation of Dynamics)

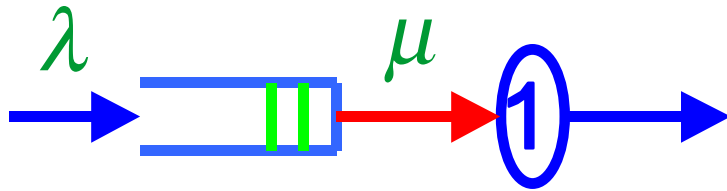


$$\rho = \lambda = 0.9524$$

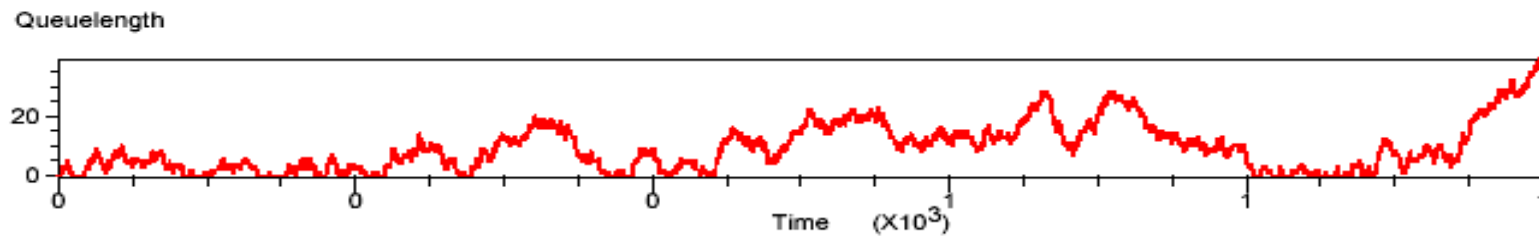
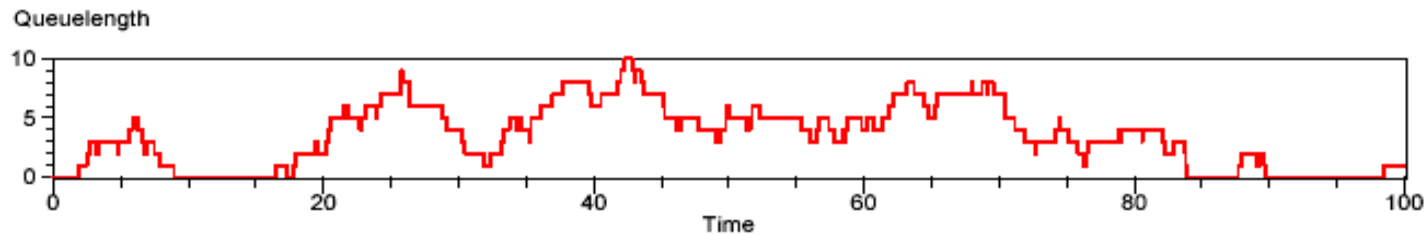


M/M/1 Queue

(Simulation of Dynamics)

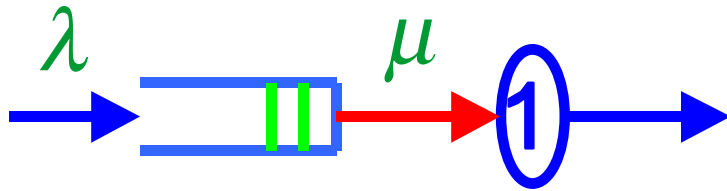


$$\rho = \lambda = 0.9524$$

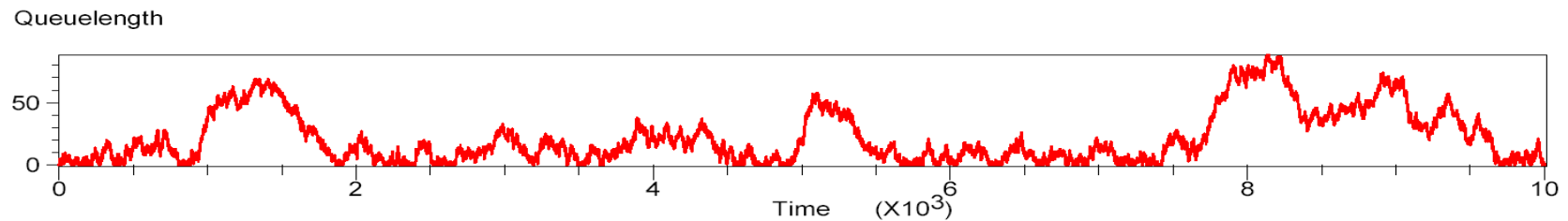
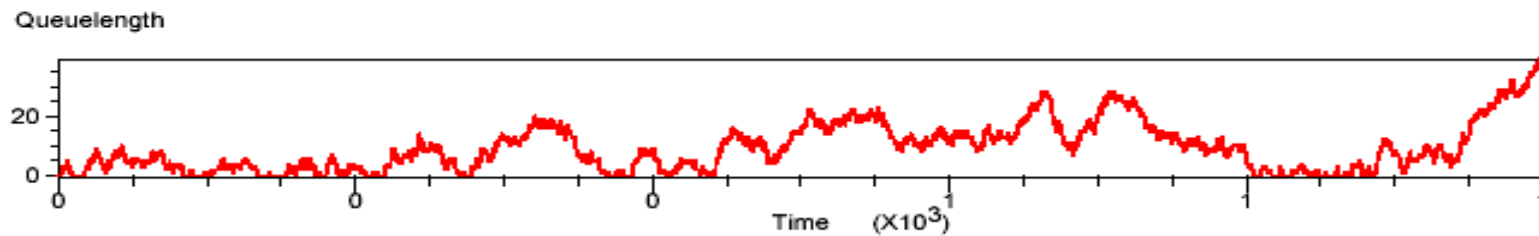
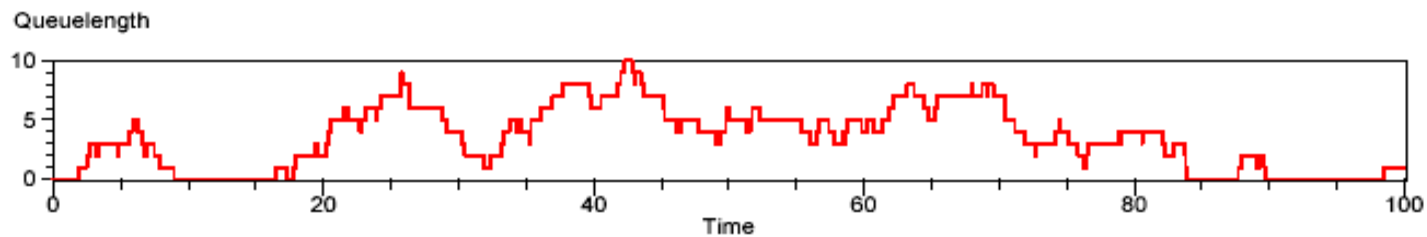


M/M/1 Queue

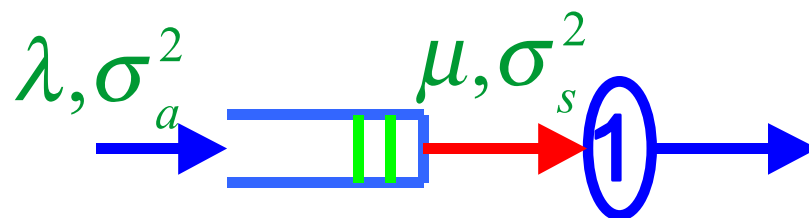
(Simulation of Dynamics)



$$\rho = \lambda = 0.9524$$



GI/GI/1 Queue (Dynamics)



$Q(t)$ = queue length at time t

Start system empty (for simplicity)

Theorem (A. Borovkov '67, Iglehart-Whitt '70): For $\rho \approx 1$,

$$(1 - \rho)Q(\cdot / (1 - \rho)^2) \approx Q^*(\cdot) \quad \text{where } Q^*(\cdot)$$

is a one-dimensional reflecting Brownian motion

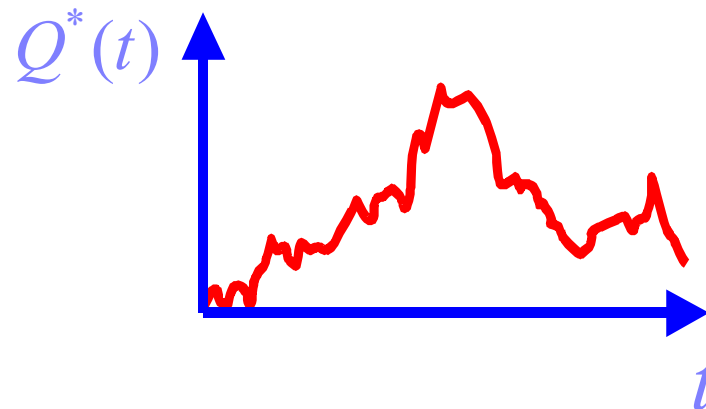
with drift $-\mu$ and variance parameter $\lambda^3 \sigma_a^2 + \mu^3 \sigma_s^2$

One-dimensional Reflecting Brownian Motion

$$Q^*(t) = X^*(t) + Y^*(t)$$

$$Y^*(t) = \sup \{ -X^*(s) : 0 \leq s \leq t \}$$

X^* = Brownian motion

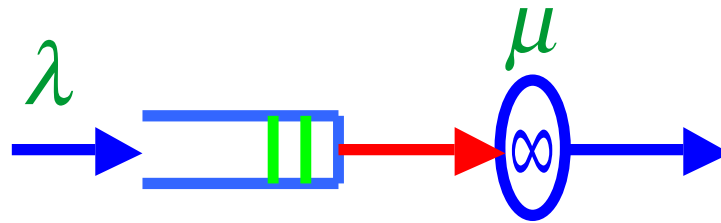


SIMPLE BIOCHEMICAL MODEL

SIMPLE BIOCHEMICAL REACTION SYSTEM



M/M/ ∞ QUEUE



$$\begin{aligned} Q(t) &= \text{number of molecules of B present at time } t \\ &= Q(0) + A(t) - S(T(t)) \end{aligned}$$

$$T(t) = \int_0^t Q(s) ds$$

A – Poisson process of rate λ

S – Poisson process of rate μ

FLUID MODEL

$$\frac{dq}{dt} = \lambda - \mu q$$

$$q(t) = \frac{1}{\mu}(\lambda + a \exp(-\mu t))$$

