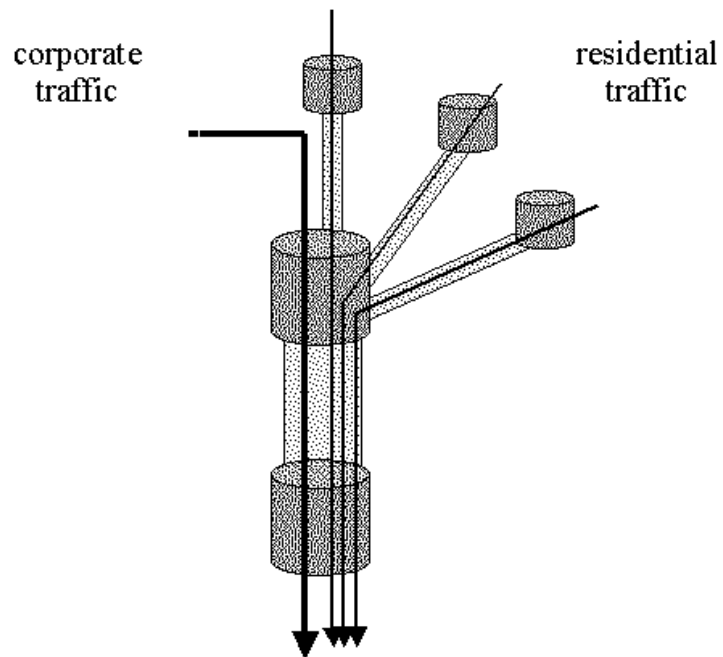
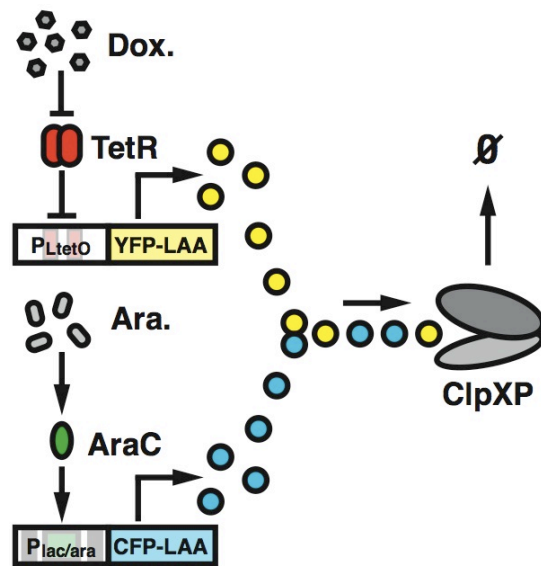


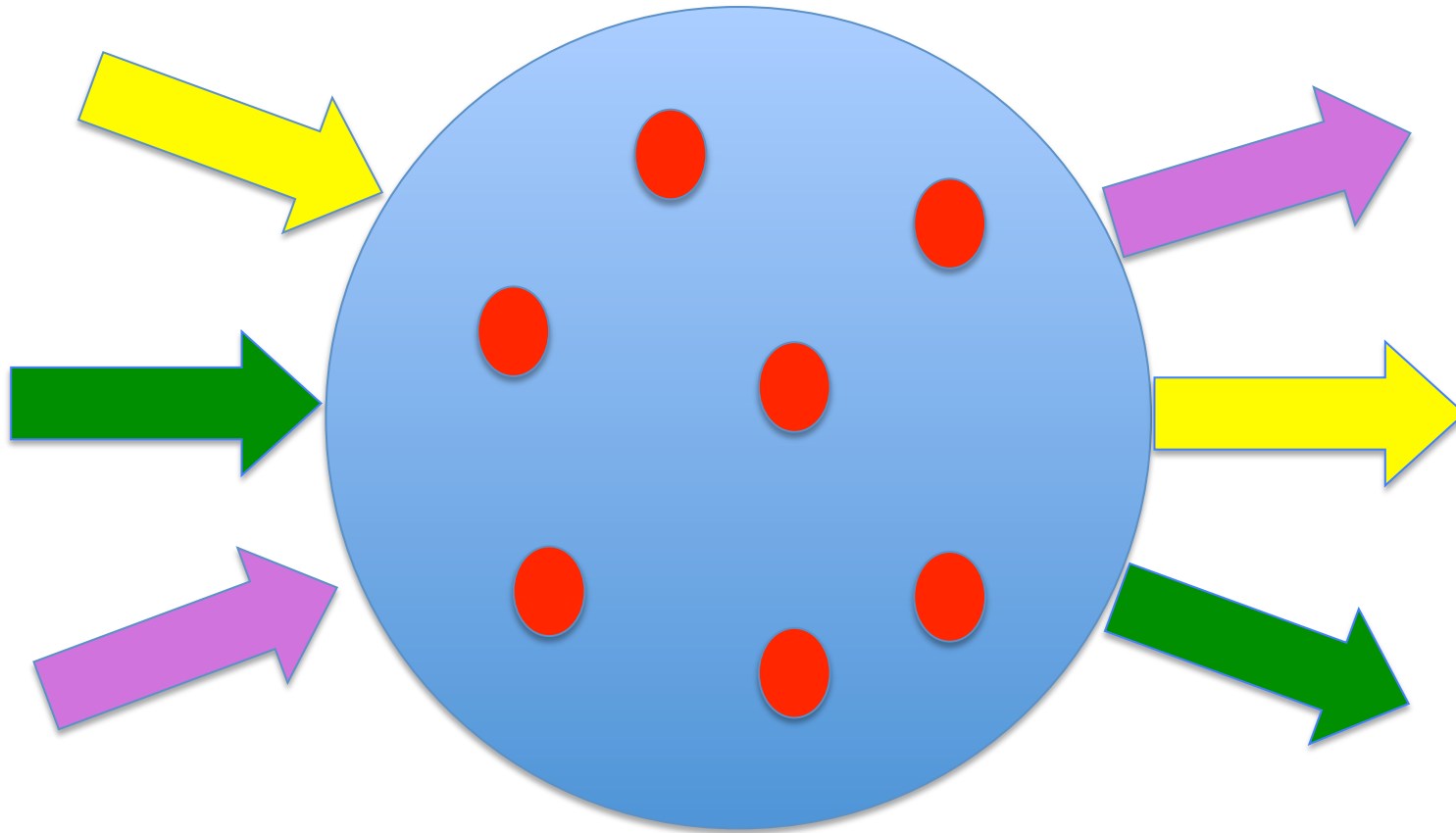
STOCHASTIC NETWORKS

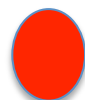


Ruth J. Williams

University of California, San Diego

STOCHASTIC PROCESSING NETWORK



 **Processing resource**

FEATURES

LIMITED RESOURCE CAPACITY → QUEUES

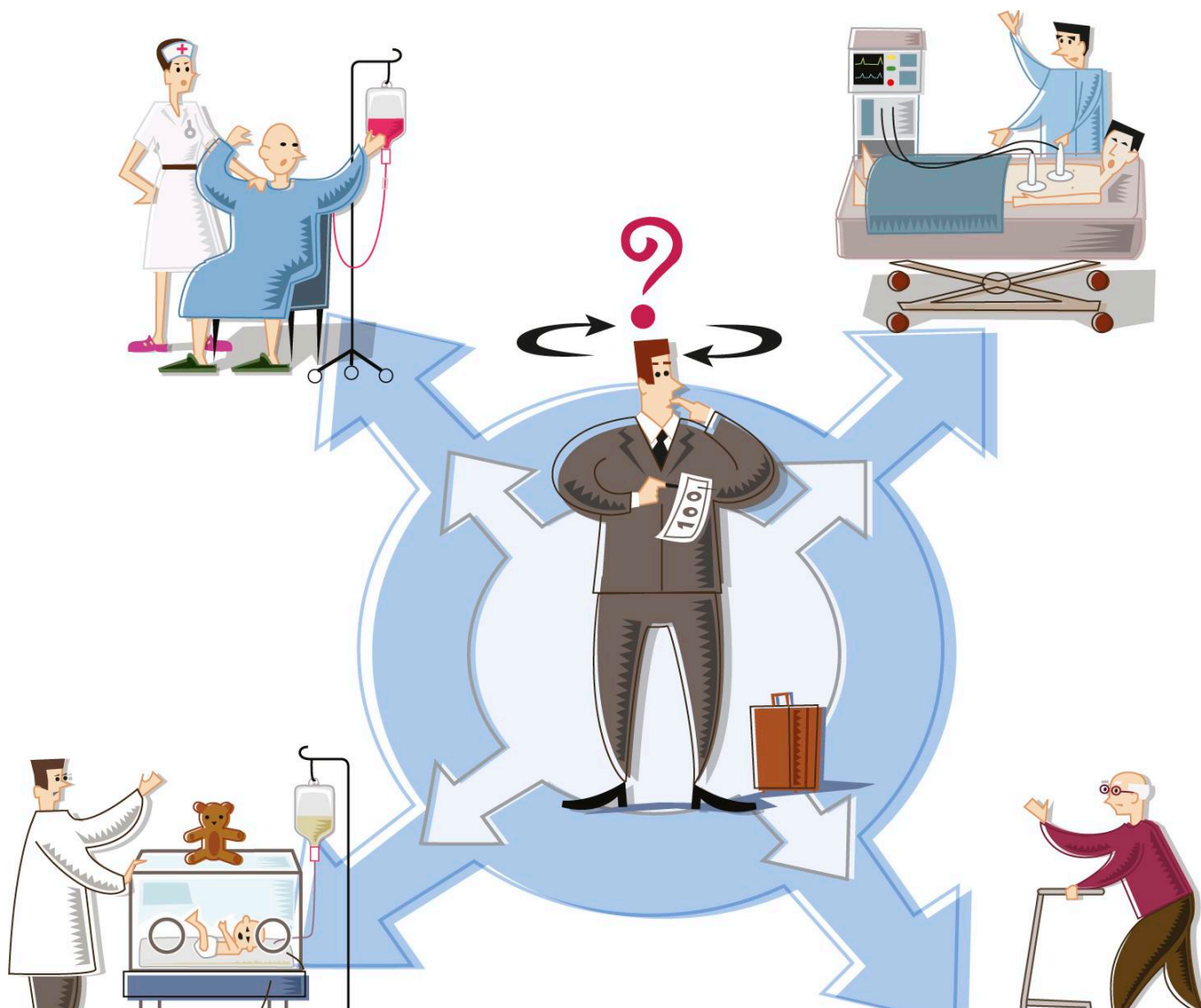


Photograph courtesy of Ilze Ziedins

EXAMPLES OF QUEUES

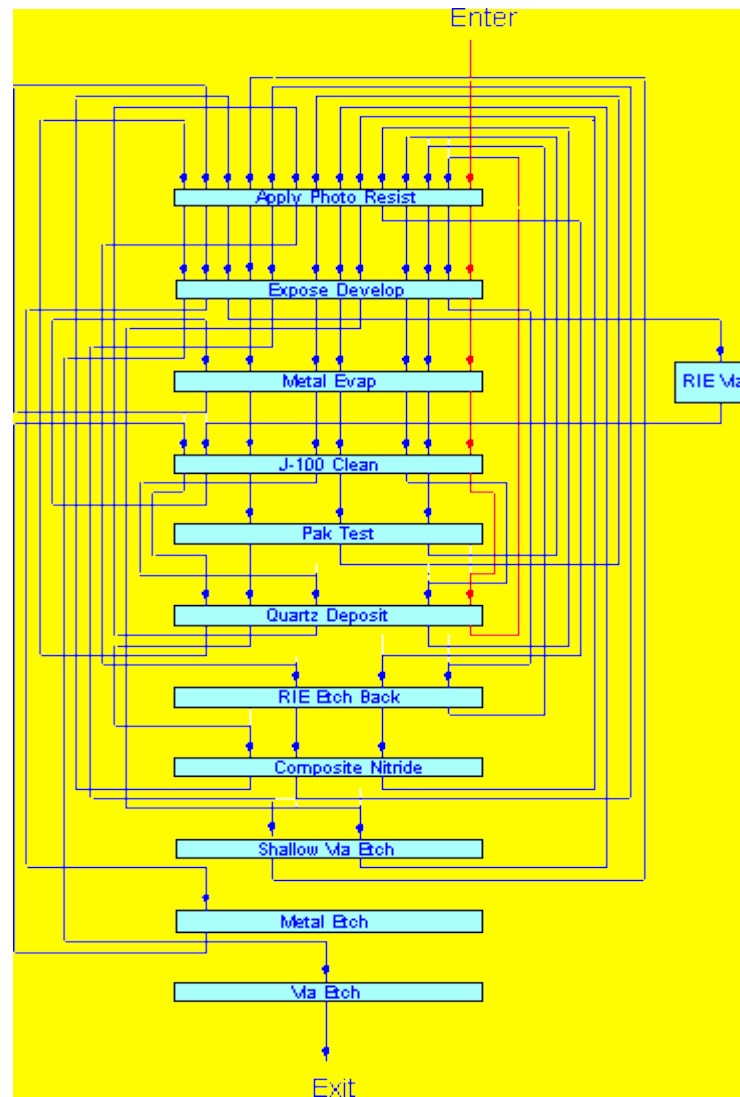
- Customers waiting in supermarket lines
- Cars waiting at a traffic light
- Patients waiting in an emergency department
- Clients waiting for service by a telephone agent
- Parts waiting to be assembled into finished products
- Packets waiting to be transmitted through a router in the Internet
- Molecules waiting for processing by an enzyme

RESOURCE SHARING



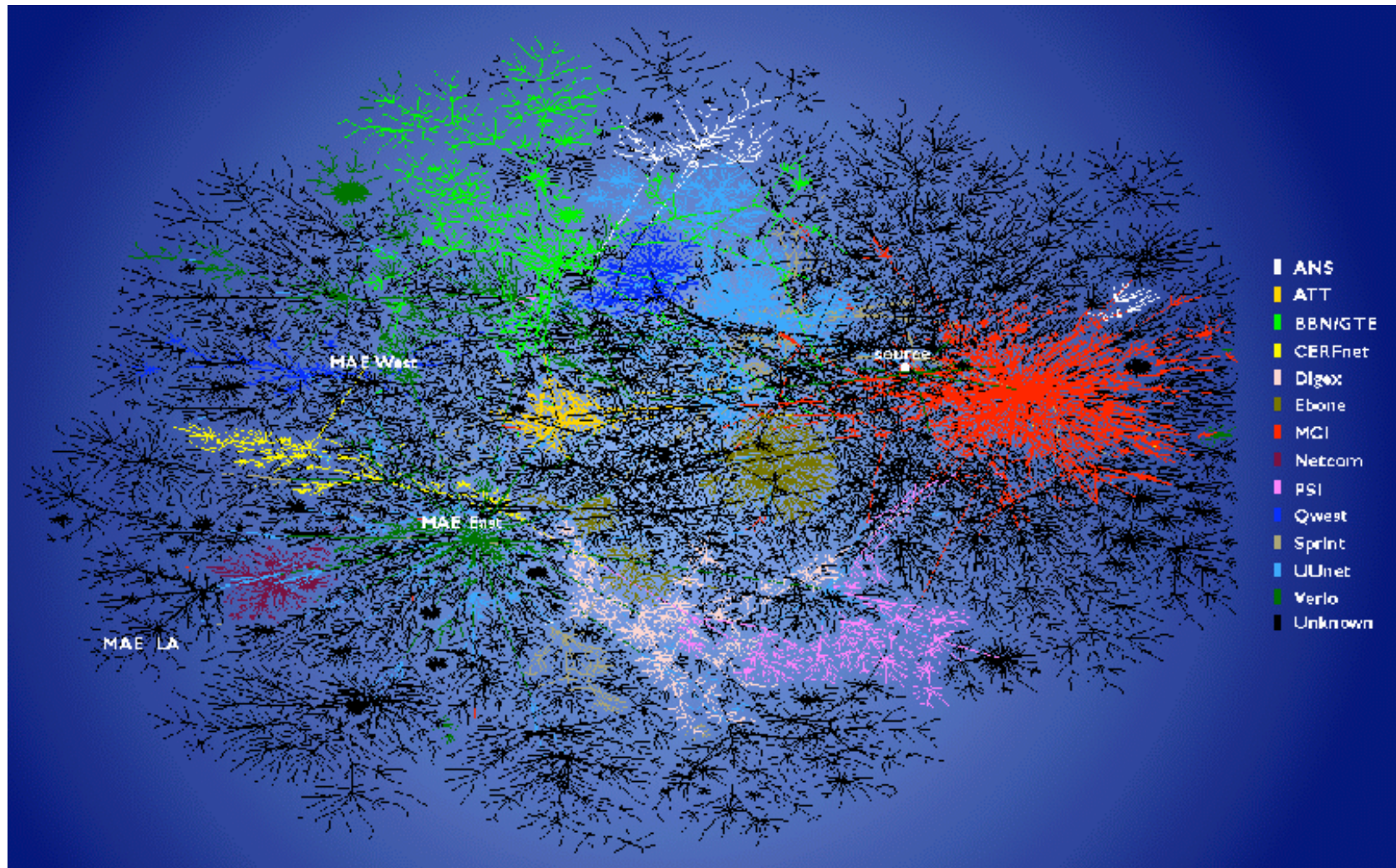
APPLICATIONS

SEMICONDUCTOR MANUFACTURING



Courtesy of P. R. Kumar

INTERNET



K C Claffy <http://www.caida.org/publications/>

TRANSPORTATION NETWORKS

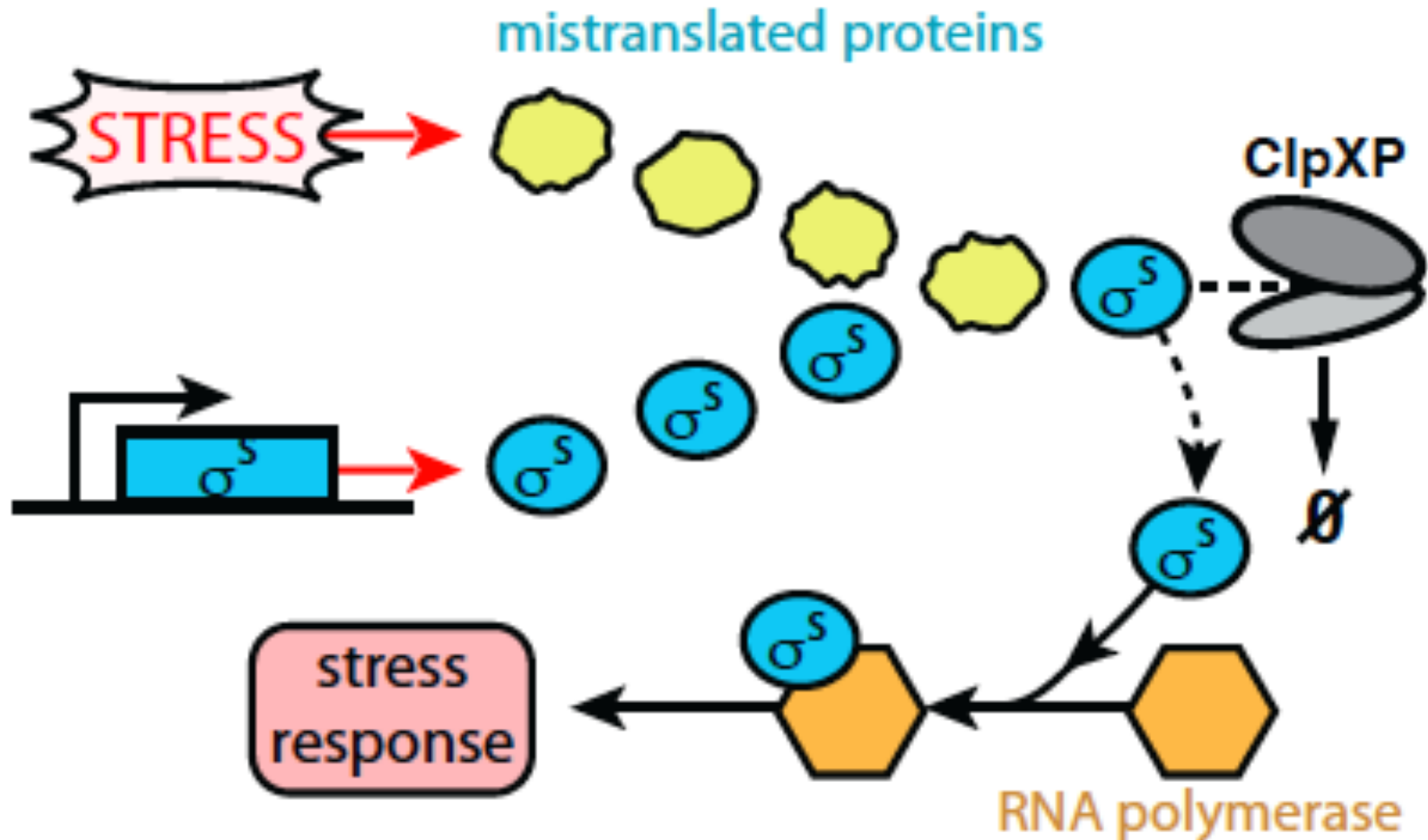


New Zealand Transport Authority

CALL CENTERS



BIOLOGICAL NETWORKS



STOCHASTIC MODELS

A HIERARCHY OF MODELS

- Microscopic models – usually stochastic, detailed dynamic models; sometimes these are Markov processes but not always; often hard to analyze
- First order deterministic ordinary differential equation models often with discontinuous dynamics – fluid models
- Second order stochastic differential equation models – often diffusion models

Important subtheme: interplay between levels of modeling

MATHEMATICAL TOOLS

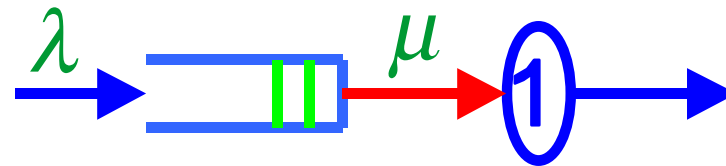
MATHEMATICAL TOOLS

- Fundamental stochastic processes
- Path spaces for stochastic processes
- Weak convergence of stochastic processes
- Invariance principles
- Stationary distributions for Markov processes
- Reversibility and quasireversibility

A SIMPLE EXAMPLE:

SINGLE SERVER QUEUE

M/M/1 Queue



- Poisson arrivals at rate λ (independent of service times)
- i.i.d. exponential service times mean $1 / \mu$
- FIFO order of service, infinite buffer
- Traffic intensity $\rho = \lambda / \mu$
- Queue length is a birth-death process (Markov)
- Positive recurrent (stable) iff $\rho < 1$
- Stationary distribution $\pi_i = \rho^i (1 - \rho), \quad i = 0, 1, 2, \dots$
- Mean steady-state queue length
$$L = \rho / (1 - \rho) = \lambda W \quad (\text{Little's law})$$

GENERALIZATION

- More general independent interarrival and service times: GI/GI/1
- Poisson processes replaced by renewal processes

QUEUE LENGTH DYNAMICS

$$\begin{aligned} Q(t) &= Q(0) + A(t) - D(t) \\ &= Q(0) + A(t) - S(T(t)) \\ &= Q(0) + A(t) - S(t) + Y(t) \end{aligned}$$

A – renewal arrival process

*S – renewal **potential** departure process*

T(•) – cumulative busy time process

APPROXIMATE DYNAMIC MODELS

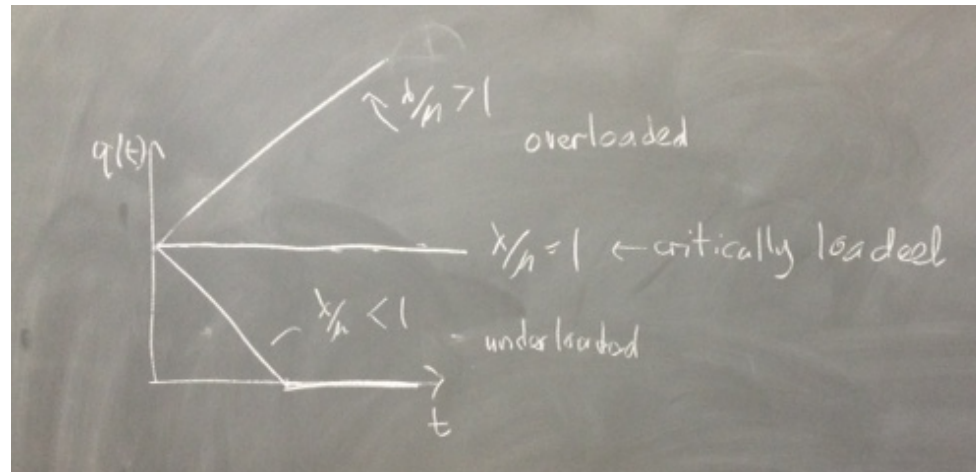
- Most stochastic network models cannot be analyzed exactly
- Consider approximate models (valid under some scaling limit, e.g., heavily loaded, many sources, many servers, large networks)
- Two main classes of approximate models:
 - Fluid models (functional law of large numbers)
 - Diffusion models (functional central limit theorem)

FLUID MODEL (FLLN)

$$\frac{dq}{dt} = \begin{cases} \lambda - \mu & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases}$$

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TWO EXTREMES

Underloaded



No queue for iPad mini in London, Nov 2, 2012
Photo by Rik Henderson

Demand < Processing Capacity
Queues are short
Little competition for service

Overloaded



Entrance to Monet's garden
Photo by Ilze Ziedins

Demand > Processing Capacity
Queues are long
Strong competition for service

TWO EXTREMES

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Queues are long
Strong competition for service*

*Balance point
(critically loaded)*