

MATH 280C**HOMEWORK #4, DUE MAY 4, 2011.**

Please hand in the starred problems. In the following, all random variables are defined on the probability space $(\Omega, \mathcal{F}, P)$.

1. Let B be a standard one-dimensional Brownian motion. Let $X(t) = B(t) - tB(1)$ for all $0 \leq t \leq 1$. The process X is called a *Brownian bridge*.

(a*) Show that $\{X(t) : 0 \leq t \leq 1\}$ is a Gaussian process and compute its covariance function $R(s, t) = \text{Cov}(X(s), X(t))$ for $0 \leq s, t \leq 1$.

(b) Show that for $0 < t_1 < t_2 < \dots < t_n < 1$, the joint distribution of $(B(t_1), B(t_2), \dots, B(t_n))$ given $|B(1)| \leq \epsilon$, converges to the joint distribution of $(X(t_1), \dots, X(t_n))$, as $\epsilon \downarrow 0$.

2*. Fractional Brownian motion with a parameter $H \in (0, 1)$ is a Gaussian process $\{X(t), t \geq 0\}$ that starts from zero, has expectation zero for all time and has the following covariance function:

$$R(s, t) = E[X(t)X(s)] = \frac{1}{2} \left(t^{2H} + s^{2H} - |t - s|^{2H} \right),$$

for all $s, t \geq 0$. Prove that there is a continuous modification of X . (Extra credit: In fact, there is a modification of X whose paths are Hölder continuous with exponent $H - \epsilon$ for all $\epsilon \in (0, H)$.)