

MATH 280C

HOMEWORK #2, DUE APRIL 22, 2015.

Please hand in the starred problems. In the following, all random variables are defined on the probability space $(\Omega, \mathcal{F}, P)$.

1*. Suppose that $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ is a filtered probability space. Suppose that S and T are stopping times on this space. Prove that

(a) $S \wedge T$ and $S \vee T$ are stopping times.

(b) if $S \leq T$, then $\mathcal{F}_S \subset \mathcal{F}_T$.

Now suppose that $\{T_n\}_{n=0}^\infty$ is a sequence of stopping times and that $\{\mathcal{F}_t, t \geq 0\}$ is right continuous. Prove that

(c) $\inf_n T_n$, $\sup_n T_n$, $\liminf_n T_n$ and $\limsup_n T_n$ are also stopping times.

(d) if $T_{n+1} \leq T_n$ for all n , then for $T = \inf_n T_n$, $\mathcal{F}_T = \cap_n \mathcal{F}_{T_n}$.

2*. Suppose that $\{X_t, t \geq 0\}$ is a continuous adapted process defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ where $\{\mathcal{F}_t\}$ is right continuous. Let A be a closed set in $(-\infty, \infty)$. Prove that

$$D_A = \inf\{t \geq 0 : X_t \in A\}$$

is a stopping time. *Hint:* for each positive integer n , $A^n = \{x : \text{dist}(x, A) < 1/n\}$ is open.