

MATH 280A

HOMEWORK #3, DUE OCTOBER 27, 2010.

1*. A probability space $(\Omega, \mathcal{F}, P)$ is said to be complete if any subset of a set $F \in \mathcal{F}$ with $P(F) = 0$ is also in $\mathcal{F}$.

Given a probability space $(\Omega, \mathcal{F}, P)$, a null set in $\mathcal{F}$ is a set whose probability is zero. Let $\mathcal{N}$ denote the collection of sets in Ω that are subsets of null sets. Let $\overline{\mathcal{F}} = \{A \subset \Omega : A \Delta B \in \mathcal{N} \text{ for some } B \in \mathcal{F}\}$. For any $A \in \overline{\mathcal{F}}$, define $\overline{P}(A) = P(B)$ when $A \Delta B \in \mathcal{N}$ for some $B \in \mathcal{F}$.

Prove that $\overline{\mathcal{F}}$ is a σ -algebra containing $\mathcal{F}$ and that $\overline{P}$ is a probability measure on $\overline{\mathcal{F}}$. Prove that $(\Omega, \overline{\mathcal{F}}, \overline{P})$ is complete.

2. Suppose that $(\Omega, \mathcal{F}, P)$ is a complete probability space. Let $\mathcal{N}_0$ denote the null sets in $\mathcal{F}$. Let $\mathcal{G}$ be a σ -algebra contained in $\mathcal{F}$. Define $\mathcal{G}^\sim = \{A \in \mathcal{F} : A \Delta B \in \mathcal{N}_0 \text{ for some } B \in \mathcal{G}\}$. Prove that $\mathcal{G}^\sim$ is a σ -algebra. It is called the augmentation of $\mathcal{G}$ with respect to $(\Omega, \mathcal{F}, P)$.

Chapter 3 of Resnick, Exercises 1, 3*, 5*.