

Math 194: Homework 6, due Wednesday February 28, 2007.

3. This exercise will require you to obtain historical data on the daily closing price of Cisco stock. Such data can be readily obtained from the Yahoo finance web site (a link to this is available from the links to related web sites on the Math 194 web site).

Consider a binomial model with $p = 0.5$, i.e., the stock price process for $t = 0, 1, \dots, T$ is given by

$$S_t = S_{t-1}\xi_t, \quad t = 1, \dots, T,$$

where S_0 is a positive constant, $\{\xi_t, t = 1, \dots, T\}$ is a sequence of i.i.d. random variables with

$$P(\xi_t = u) = p, \quad P(\xi_t = d) = 1 - p,$$

$0 < d < 1 + r < u$, and r is the risk-free interest rate. Let

$$X_t = \frac{S_t - S_{t-1}}{S_{t-1}} = \xi_t - 1, \quad t = 1, \dots, T.$$

Then, with $p = 0.5$, the mean and variance of X_t are given by

$$\mu = E^p[X_t] = pu + (1 - p)d - 1 = \frac{u + d}{2} - 1, \quad (1)$$

$$Var = E^p[(X_t - \mu)^2] = \frac{(u - d)^2}{4}, \quad (2)$$

respectively.

Suppose that you have a sample of closing stock prices $\tilde{S}_0, \tilde{S}_1, \dots, \tilde{S}_T$ for times $t = 0, 1, \dots, T$. For $t = 0, 1, \dots, T$, let

$$\tilde{X}_t = \frac{\tilde{S}_t - \tilde{S}_{t-1}}{\tilde{S}_{t-1}}.$$

Then, the usual (unbiased) statistical estimators for μ and Var are given by

$$\bar{\mu} = \frac{1}{T} \sum_{t=1}^T \tilde{X}_t, \quad \overline{Var} = \frac{1}{T-1} \sum_{t=1}^T (\tilde{X}_t - \bar{\mu})^2.$$

- (a) Compute these estimators using the daily closing stock price data for Cisco (ticker symbol CSCO), commencing with January 18, 2007 and ending with February 22, 2007.

You can obtain this historical data in downloadable Excel spreadsheet form from the Yahoo finance website <http://finance.yahoo.com/>. Once there, type in the ticker symbol CSCO, select “Historical Prices” in the left side column. Then type in the range of dates for which you want data and select “Daily”, then click on “Get Prices”. The historical data for the

range of dates will be displayed and can be saved in spreadsheet format as a .csv file by clicking on “Download Spreadsheet Format” at the bottom of the table. Once you have saved the data to a file, you can open that file using Excel (note that it has a .csv extension, not a .xls extension). You can then use the built in Excel functions Average and Var to compute the above estimators for a column of data – you can also use Stdev instead of Var to obtain the square root of Var. Here, the natural choice for one unit of time is one trading day (ignore non-trading days such as weekends and holidays in counting time).

Using these estimates in place of μ and Var in (1) and (2) respectively, determine estimates for u and d .

- (b) Now consider the binomial model with these estimated values of u and d .

Based on this binomial model, find the arbitrage free price on February 23, 2007 for

- (i) a European call option based on CSCO stock with a strike price of \$25 and an expiration month of March,
- (ii) an American put option based on CSCO stock with a strike price of \$25 and an expiration month of March.

Notes:

(N1) To estimate the daily risk-free interest rate r , use the 3-month T-bill rate on February 23, 2007 (take this to be 5.03% per annum). As an approximation, assume there are $65 = 13 \times 5$ trading days in a 3-month period, set $(1 + r)^{65} = 1 + \frac{0.0503}{4}$ and solve for r .

(N2) Recall that all market-traded options (such as we are considering) effectively expire on the third Friday of their month of expiry (we say effectively because options cannot be traded on a Saturday). For March 2007, the third Friday is March 16, 2007.

(N3) To compute the arbitrage free prices, you will probably want to use an Excel spreadsheet program, such as you have been working with on previous homework assignments. You will need to draw a bigger binomial tree with more steps in it for this – in particular, you will need to start further to the right in order to have room for your tree to spread.)

- (c) Comment on your findings, especially in comparison to the option prices listed for CSCO corresponding to the close of trading on February 23, 2007 (you can find option prices on the Yahoo website).