Where do we go from here?

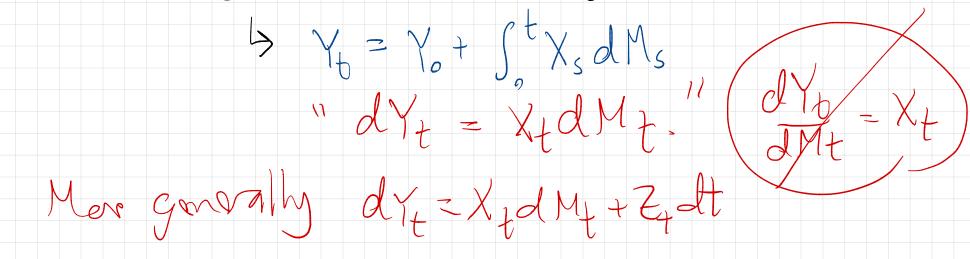
This course has been an introduction to modern probability theory. We have barely scratched the surface of Probability, and with few exceptions (like Stein's method), we've introduced theory that has been well-understood and standardized since the 1940s (or earlier)

It would be difficult le give a "comprehensive list" of probability topics out there today.

Instead, I wanted to leave you with thoughts on a few netural next steps for subjects / courses that follow naturally from where we've ended up.

- * Continuous-Time (Sub) Martingales (Xt) +> 0 6 L¹, E[Xt-Xs [Fs]= 0 Vs<t
 - La Convergence theorems (Lim Xt, lim Xt under L'-bounded, unif integrability)
 - La Optilenal Stopping, Optional Sampling La Maximal / LP inequalities

 - La Regularization ; filtration augmentation and right continuity La Applications to (more) properties of Brewnian motion.
 - (This could have been done in this course, with 2-3 more weeks)
- * Stochastic Integration
 - Martingale (ME) t=0; progressive process (Xt) t=0





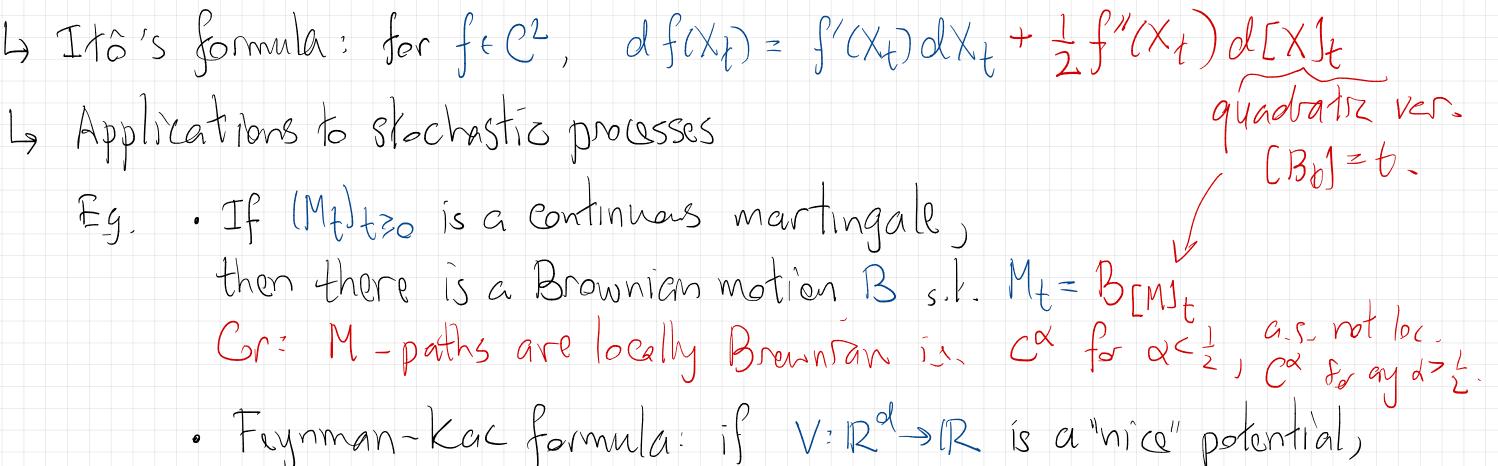


Ly Itô's formula: for $f \in C^2$, $d f(X_t) = \int (X_t) dX_t + \frac{1}{2} \int (X_t) d[X_t]_t$

- - - · Frynnan-Kac formula: if V:12 ->12 is a "nice" potential, the PDE
 - $\partial_t u = \pm \Delta u V \cdot u$ $u(c, \infty) = f(\infty)$

has unique solution

unique somtion $fV(B_5)dS_7$ $u(x,t) = E^{20}[f(B_2)e^{-5V(B_5)}dS_7.$



* Stochastic Differential Equations

dXt = 5(t,Xt) dBt + m(t,Xt) dt 2 Stohastic ODE

(Fairly well-defined, well understand existence / uniqueness)

Stochastic PDEs: much harder, even to make sense et.

La Connections la random surfaces,

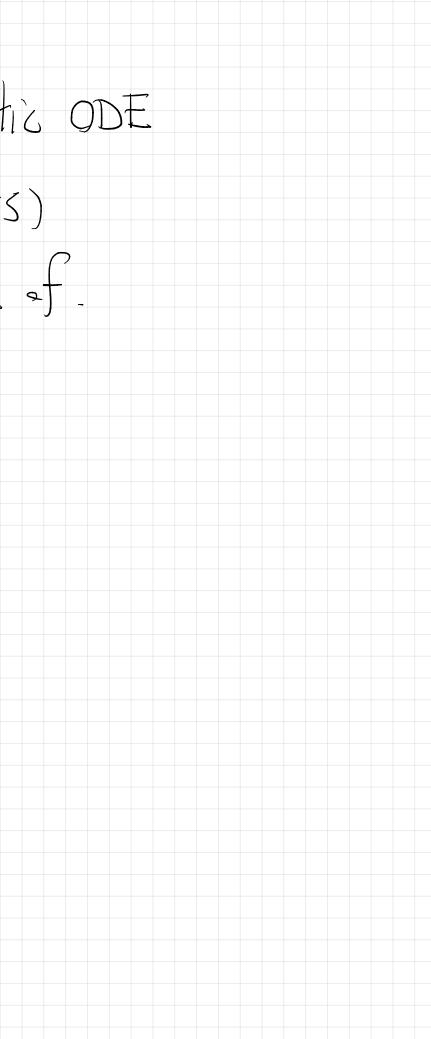
random geometry, statistical physics, ---

* Feller - Markov Processes

7 Lo Markov processes in locally compact metric space S take a Funct-brevisis carse: 2. IQ+F-Film > 0 as the for fe C. (s) 2. $\|Q_t f - f\|_{\infty} \rightarrow 0$ as the for $f \in C_0(S)$.

Eg. Brennen motor, Person process.

Continuous - time homogéneous Masker precesses with generators Q6= eta A densely-defined in Cols).



Then there are lots of topics that Continue from points throughout this course

- * Renewal processes, birth and death processes
- * Queueing systems
- * Large Deviations
- * Stein's Method
- * Entropy
- * Stable distributions
- * L'evy processes

And there are probability and related fields that are very "hot" right now, including = * Random Matrix Theory (and Free Probability)

* Random Graphs and Nethorks

* Random fields and surface growth (KPZ, SLE)

And ...