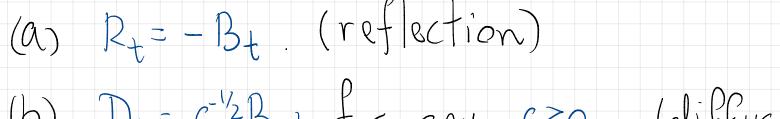
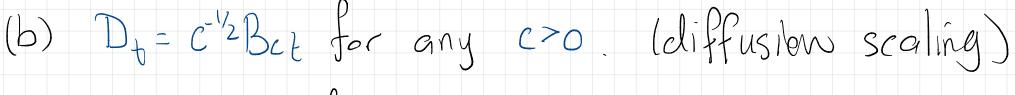
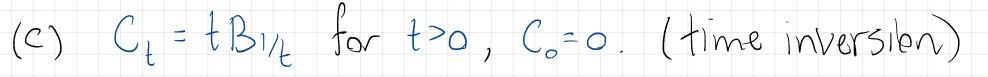


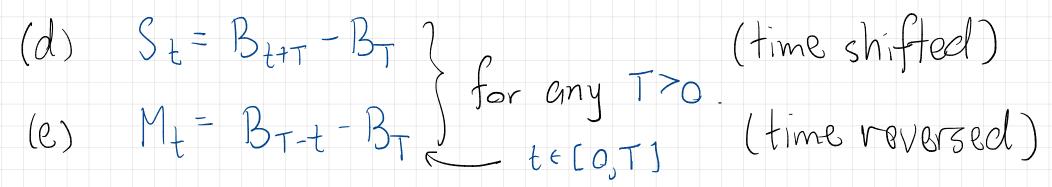


are also Brownian motions





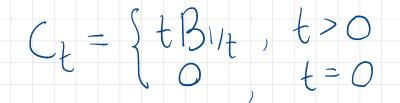






2. In all cases, it is easy to check that the process is Gaussian, and centered. In (G), (b), (d), (e) continuity is also clear, and so the result follows by calculating the Covariance, which is routine.

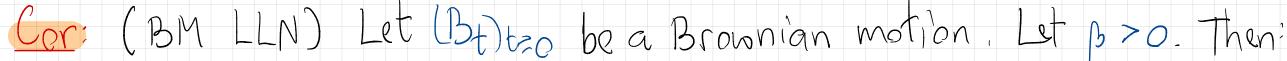
We'll focus en time inversion (c).





· Mean and Covariance:

. Continuity: (only t=0 is unclear)



 $\begin{array}{c|c} limsup |B_{1}| = \begin{cases} 0 & \text{if } \beta > \frac{1}{2} & q.s. \\ t \rightarrow col & t^{\beta} & 0 & \text{if } c < \beta < \frac{1}{2} & q.s. \end{cases}$ 

## Pf. Let Ct=tB14. Since C. is a Brownian motion, (Ct)tec, 1 is a.s. Ca for a < 1

OTOH, per [Lec 54.2],  $\lim_{s \to 0} \sup_{s \neq 0} |C_s| = co q.s.$  if  $\alpha > \frac{1}{2}$ 



The exact vote of divergence of Brownian motion as to as is known.

Theorem (Khinchin, Kolmogorov; Hartman-Wintner)

The Law of the Iterated Logarithm

 $\begin{array}{c} limsup \pm B_t \\ t \rightarrow \infty \end{array} = 1 \quad a.s. \end{array}$ 

Using Donsker's CLT, this can be used to show = if (Xn)nzi are iid L<sup>2</sup> standardized,

 $\lim_{n \to \infty} \sup_{\sqrt{2n \log(\log n)}} = 1 \quad a.s.$ 

Note: by the classic CLT,