Stopping Times

Let (SL, F, {Fn}neN, P) be a fittered probability space.

A rundom variable T: S-> Nu(+co) is a (discrete) stopping time F

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Equivalently: iff the process

Eq. If $(X_n)_{n\in\mathbb{N}}$ is an adapted process in (S, B)and BC B, then $T_B = \inf\{n \ge 0: X_n \in B\}$ is a stopping time. $\{T_B \le n\}$

How about the final hitting time LB = sup{nzo: XncB}?

 $\{L_B \le n\} = \bigcup_{k \le n} \{L_B = k\}$

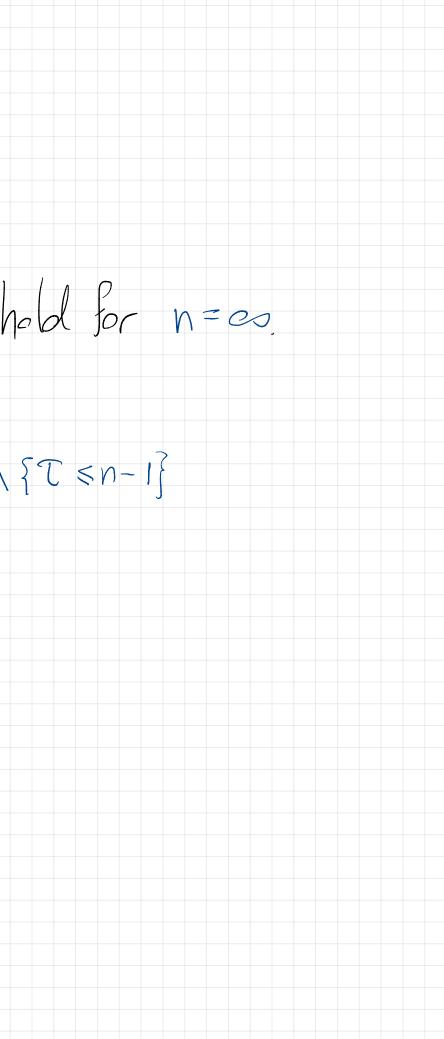




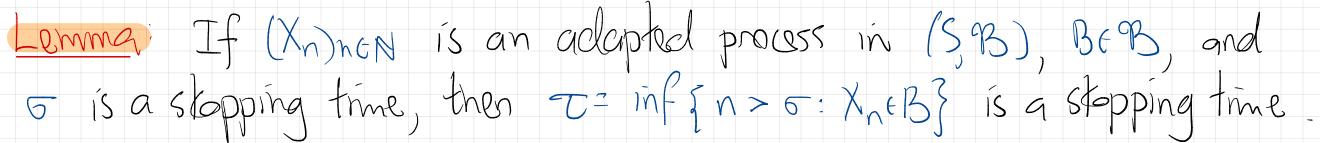
- 1. T is a stopping time, i.e. $\{T \le n\} \in \mathcal{F}_n$ $\forall n \in \mathbb{N}$ 2. $\{T > n\} \in \mathcal{F}_n$ $\forall n \in \mathbb{N}$.
- 3. $\{T=n\}\in F_n$ $\forall n\in \mathbb{N}$.
- Mereover, if any one of these conditions hold, then they also hold for n=00. Pf. (1)=(2)=>(3) follow readily from the identities $\{T > n\}^{c} = \{T \le n\} = \bigcup_{v \in V} \{C = k\}, \ \{T = n\} = \{T \le n\} \setminus \{T \le n - 1\}$
- Now, if T is a stopping time, then
 - $\{\tau < \infty\} =$
 - · . { T = 00 } =

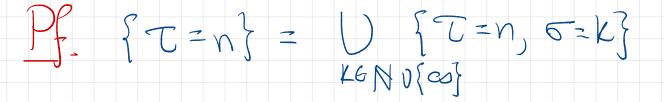


and {t>co}



Eq. we saw the first hitting time of an adapted process is a stepping time. How about the second hitting time? The billionth?





We can combine stopping times to make new ones.

Lemma If o, t, it is are stopping times, then 1. 5 ~ t, 5 ~ t, 5 + t are stopping times. 2. If it is monotone, then lim the is a stopping time.

