(Biosed) Random Walks on Z

Fix pe (0,1). Consider the Markov chain on Z with transition kernel

Let B=Z. We can use fist-step analysis

to understand TB and LB = XTB.

 $u(x) = E^{x}[h(X_{TB}): T_{B} < \infty] \quad h = 1, u(x) = P^{n}(T_{B} < \infty)$ 

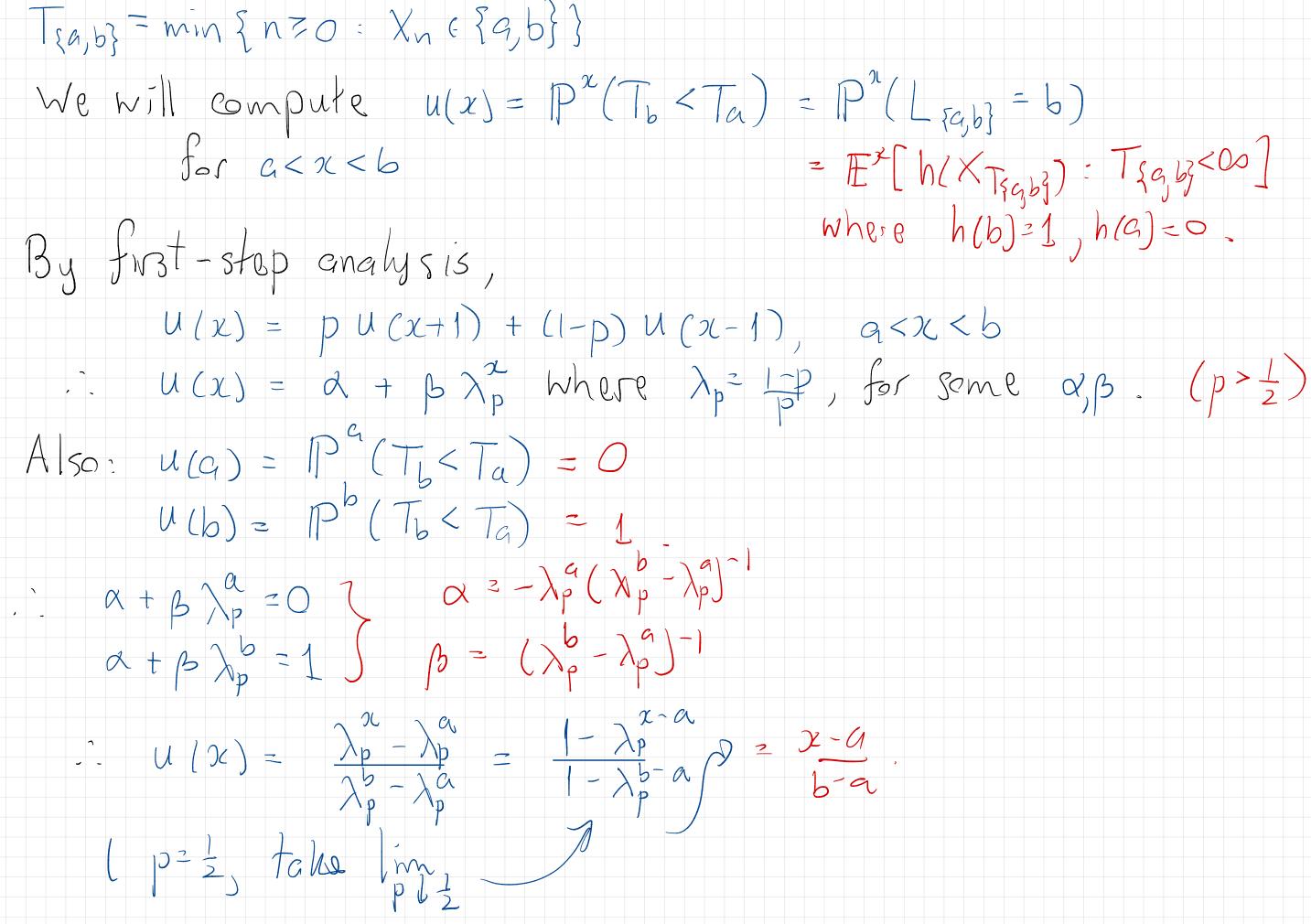
We know u = h on B, and u = Qu on  $B^{C}$ 

 $u(x) = \sum_{y} q(x,y) u(y) = pu(x+1) + (1-p)u(x-1)$ 

3-term recursion  $pu(x+1) - u(x) + (1-p)u(x-1) = 0 x \neq B$ cher\_polynemial:  $p\lambda^2 - \lambda + (1-p) = 0$ reots:  $\lambda = 1$ ,  $\lambda = \lambda_p = \frac{1-p}{p^2}$ .

 $\begin{array}{c} q(x,y) = p \rfloor y = x + i + (1 - p) \rfloor y = x - i \\ \text{id.} \stackrel{d}{=} p \delta_1 + (i - p) \delta_{-1} \\ \text{(Simple random walk is the case } p = \frac{1}{2} \\ \text{We can construct this process explicitly:} \\ X_n = \frac{1}{2} e + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \\ \text{whe can construct this process explicitly:} \\ \text{We can c$ 

General solution: for x & B,  $\lambda_p^2 \xrightarrow{FP} J$  for some  $\lambda_b \in C$  $\begin{cases} p \neq \frac{1}{2} & u(x) = \alpha + \beta \lambda_p^{\alpha} \\ p = \frac{1}{2} & u(x) = \alpha + \beta \lambda_p^{\alpha} \end{cases}$ Symmetry:  $q(-x,-y) = p \lim_{z \to y=-x+1} + (1-p) \lim_{z \to y=-x-1}$ =  $p 1_{y} = x - 1_{t} + (1 - p) 1_{y} = x + 1_{t}$ I.e. for the associated Marker chain (Xn)neN, (-Xn)neN is the "Same" chain with part-p. WLOG take  $p \in (\frac{1}{2}, 1)$ .  $n = \frac{1}{2}$   $n = \frac{1}{2}$   $n = \frac{1}{2}$ We'll consider the case B= 29,63 with a<6 As we'll see, there's a nice relationship between Trabi and Ta, Tb.



 $= E^{2}[h(X_{T_{s}}) : T_{s} + C_{s}]$ where h(b) = 1, h(c) = 0.

## Grambler's Ruin

You play a game against the House; the probability of the House winning is  $p > \frac{1}{2}$ 

Xn = House's fortune (your bases) after n plays.

 $\lambda_p = 0.89$ 

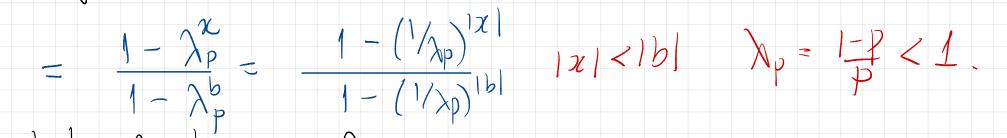
Your initial fortune is \$x (x < 0 as we're viewing from the House perspective).You play repeatedly (winning or bying \$1 each play) until you go broke  $X_n = 0$ 

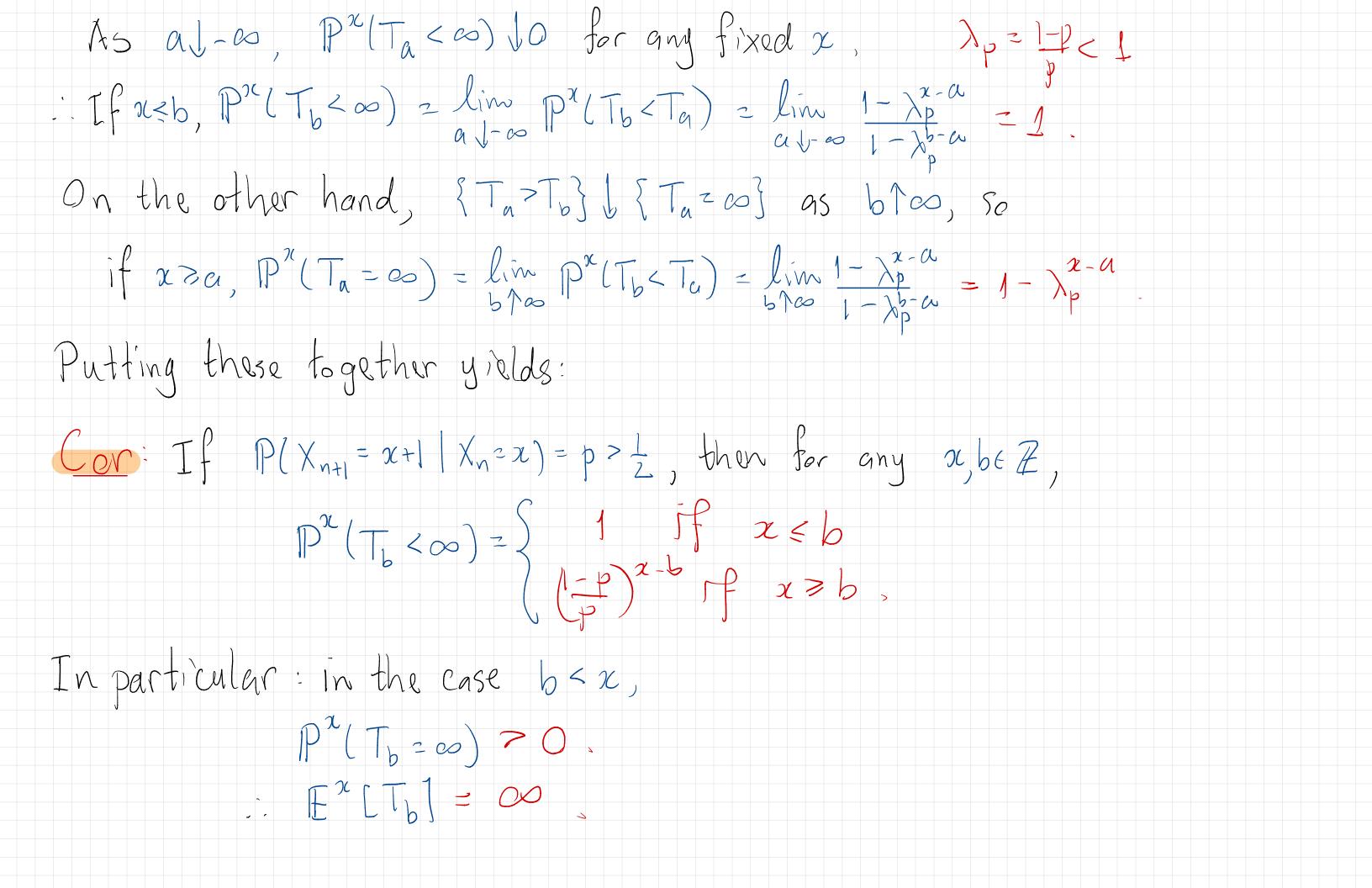
P(Yen win \$6, never going broke) = P<sup>x</sup>(Tb < To)

Eq. 161=2121 (Want to double your fortune)  $= \frac{1 - (1/\lambda p)}{1 - (1/\lambda p)} \frac{121}{236} \frac{121}{66}$ 

 $= \frac{1}{1 + (V_{\lambda p})} |x| < \lambda_{p}$ What about Just Tb?  $P^{\chi}(T_{b} < \infty)$ ?  $E^{\chi}(T_{b}]$ ?  $E_{y}$  p=0.53  $P^{\chi}(T_{b} < \infty)$ ?  $E^{\chi}(T_{b}]$ ?  $E_{y}$  p=0.53

or you win \$6 Xn=6<2<0.





## what about b > x ? Here $P^{*}(T_{b} < \infty) = 1$ . But is $E^{*}[T_{b}] < \infty ?$

A similar 1st step analysis slows that

