In a discrete (homogeneous) time, countable state space Markov chain,

a state i is called absorbing if q(i,i)=1.

q(i,j) = 2

In general, if there's any loop, quiji)>0, the Markov chain is called lazy. Assuming no absorbing states, we can define a kernel

We can describe the Markov chain with kernel q.

Propi (Jump-Hold) Let (Yn)ner be a time homogeneous Markov chain with 1-step transition Kernel q.

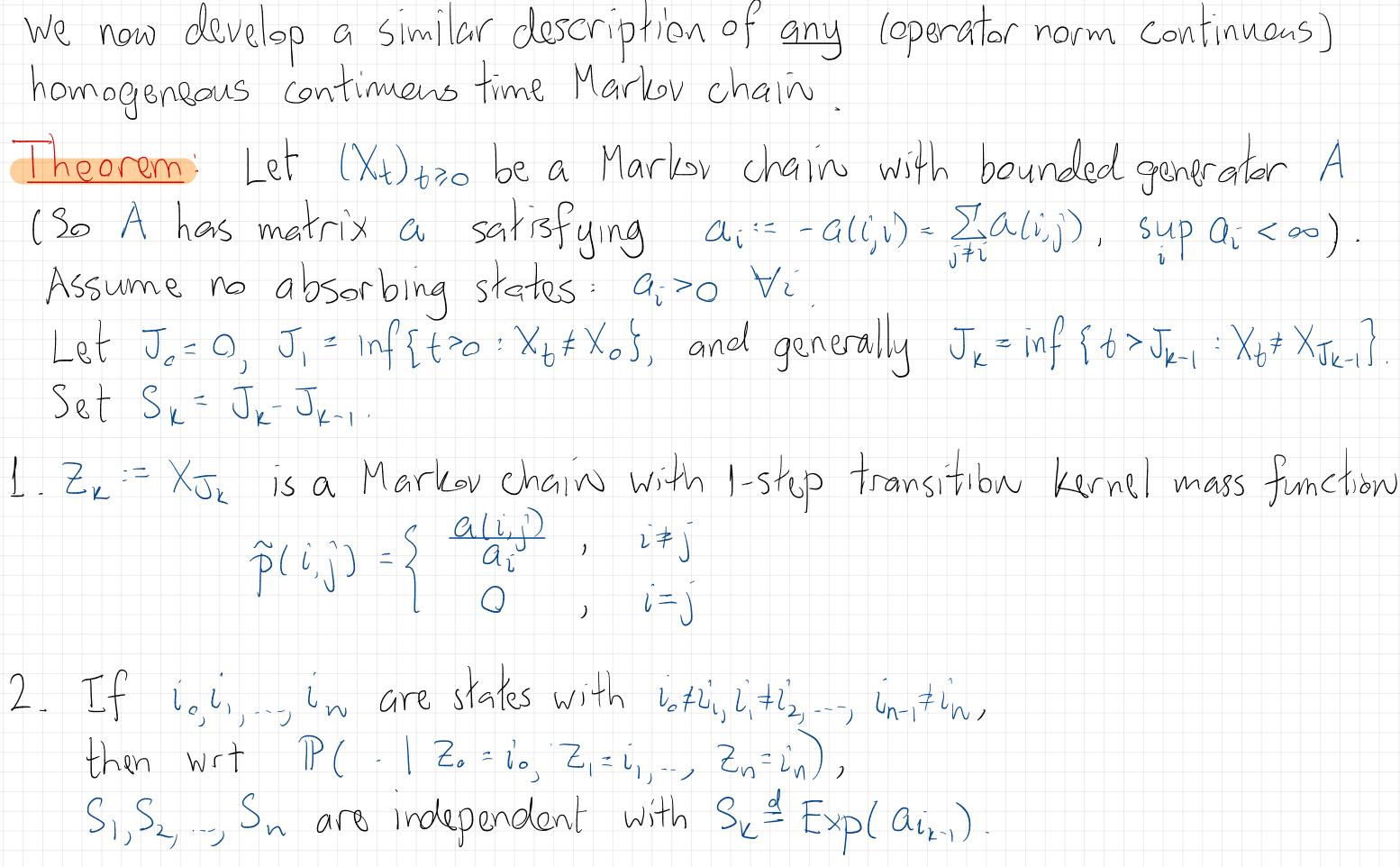
Set $T_1 = \inf\{n > 0: Y_n \neq Y_o\}, T_{k+1} = \inf\{n > T_k: Y_n \neq Y_{T_k}\}.$

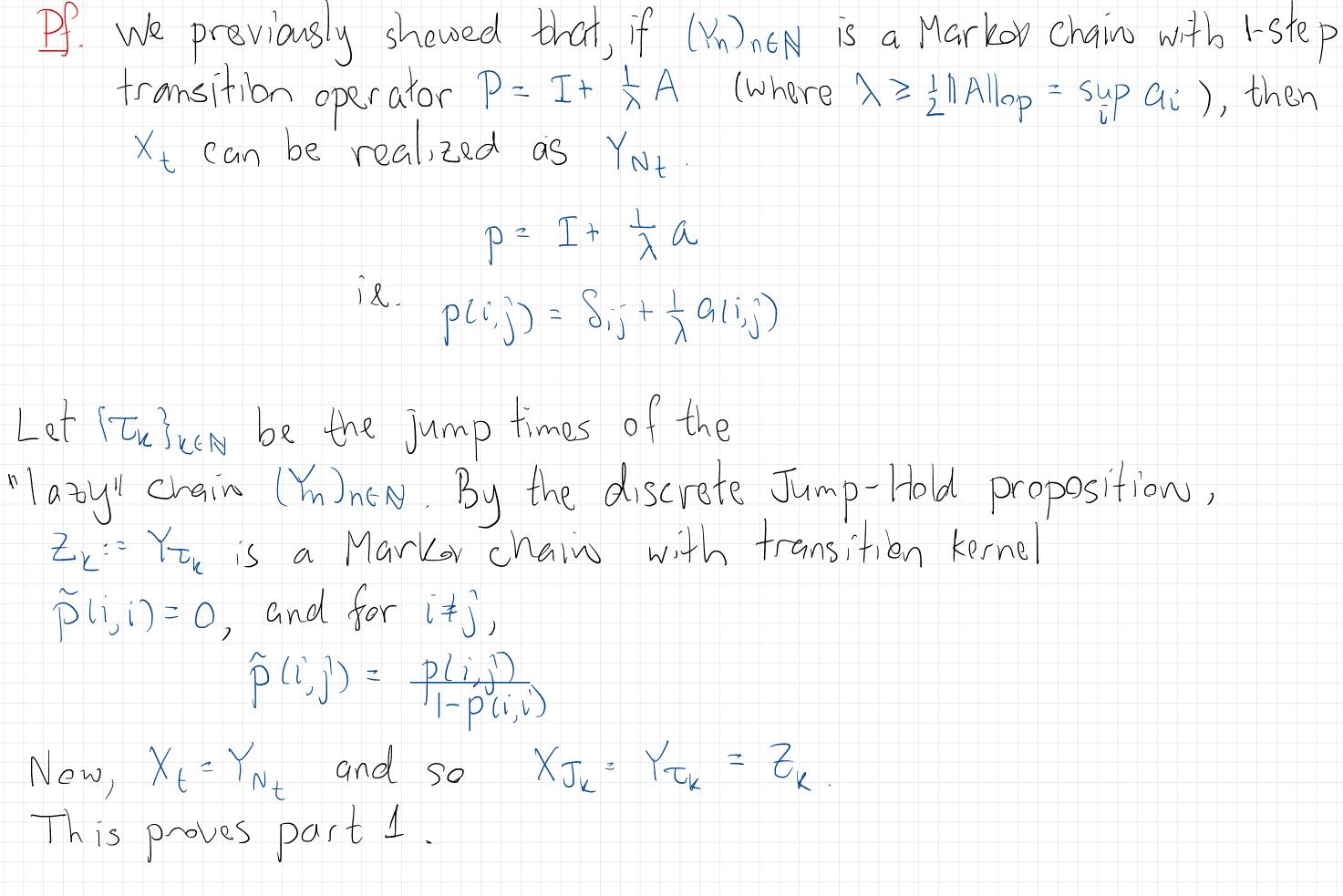
Then (ZK=YTK)KEN is a Markov chain with

I-Step transition Kernel q. Moreover, wrt. P(-1Z=i,-, Zk=ik), {5k= Tk-Tk-13k=1 are independent

PF [HW]

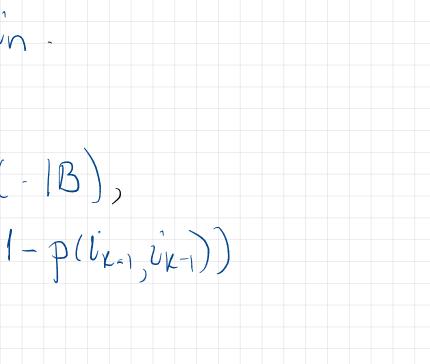






For 2, let or = TK-TK-1. Fix states ie + i, i, + i2, -, in-, + in. Let $B = \{2_0 = i_0, 2_1 = i_1, ..., 2_n = i_n\}$. By the discrete Jump-Hold proposition, relative to P(-1B), $\overline{5}_{1,-7}, \overline{5}_{n}$ are independent, with $\overline{5}_{k} \stackrel{q}{=} Greon(1 - p(i_{k-1}, i_{k-1}))$ Note that $T_k = 5, + - + 6_k$ Similarly: $J_k = (k^{th} jump time of X_t) = T_1 + T_2 + \dots + T_{t_k}$ where Te are the jump times of NE. They are = Exp(),

Lemma: Let Te = Exp() for LEN, $\{\sigma_{1}, \sigma_{n}\}$ Satisfy σ_{k}^{2} Grow (b_{k}) , and { Tesler U { 5, -, 5, } are independent. Set $W_n = T_1 + \cdots + T_n$ $GS_l = W_{\tau_l} - W_{\tau_{l-1}}$. $T_k = G_1 + \cdots + G_k$ $GS_l = W_{\tau_l} - W_{\tau_{l-1}}$. Then {S,..., Sn} are independent, with $S_{\ell} \stackrel{d}{=} E_{\times p}(b_{\ell} \cdot \lambda)$ [1/W]



and independent.

