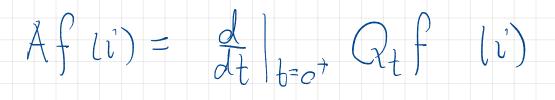


Thus, if (gt) two are Markov transition kernel mass functions on a discrete state space S, satisfying

## $\lim_{t \downarrow 1} \inf_{i \in S} q_t(i,i) = 1$

then the transition semigroup (Qt) too has a bounded generator A.

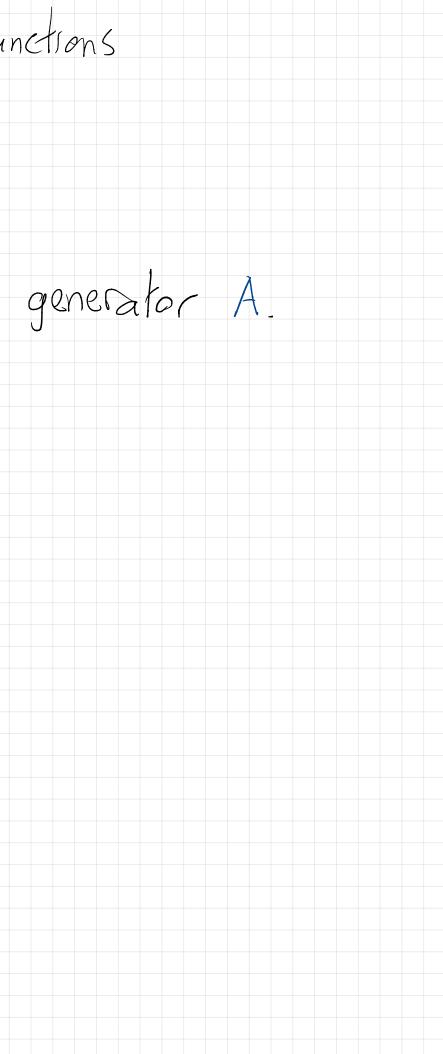
Question: What can we say about A?



This suggests that A has a matrix a

given by  $a(i,j) = \frac{d}{dt} q_t(i,j)|_{t=0^+}$ 

If S is infinite, this takes some work to prove.



First question; does A even have a matrix?

You might think this must always be true: that every (bounded) linear operator

## $T: B(S) \rightarrow B(S)$

has a matrix. Following the finite-dimensional case, we would expand

If f is a simple function, so f(j)= for all but finitely many jes,

So we would expect that T has matrix  $\Theta(i,j) =$ 

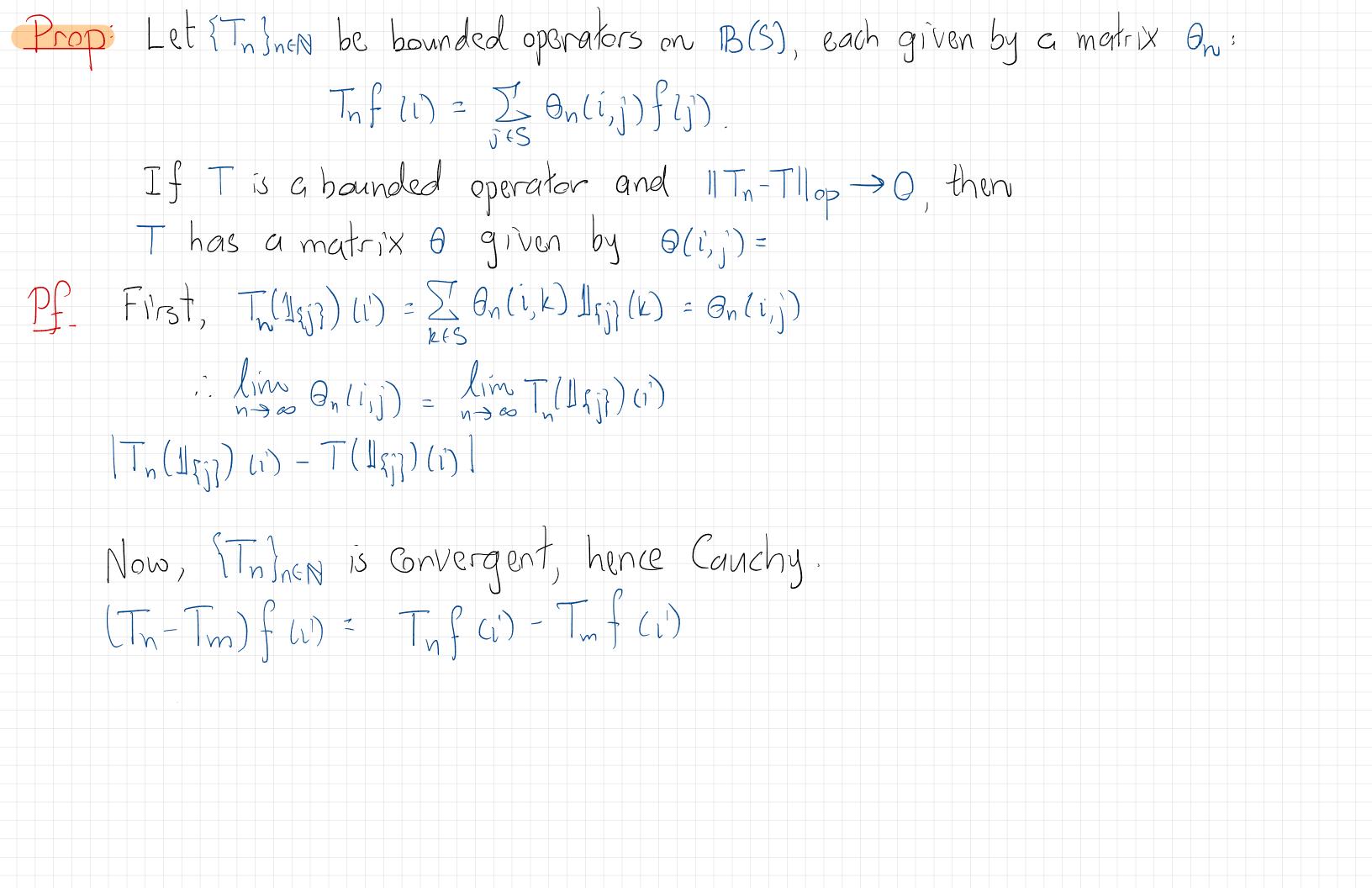
Tf =

But if f is not simple, there's no way to extend this:

even if nolles < as, we can't check if Tf(i) = < O(i,j)f(j).

Basic Problem:

Fact: I bounded operators on B(S) that have no matrix.



: The has matrix on-on,  $\|\Theta_n - \Theta_m\|_{\infty} = \|T_n - T_m\|_{op}$  $\frac{\sup 2}{i P \sum_{j=1}^{n} |\Theta_n(i,j) - \Theta_n(i,j)|}{|\Theta_n(i,j)|}$  $:= For each i, \sum_{i} |\Theta_n(i,j) - \Theta(i,j)| = \sum_{i} \lim_{m \to \infty} |\Theta_n(i,j) - \Theta_m(i,j)|$ 

As this is true for all i, taking sup . By the triangle inequality, 1101100 <00, so it defines a bounded operator  $\widehat{T}f(i) := \sum_{j} \theta(i, j)f(j).$ But then II T-Tullop = 110-Oulles Since IIT-Trillop >0, it follows that

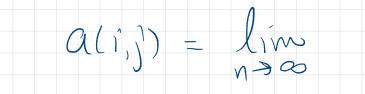
Cor: Under the continuity condition 11Qt-Illop70 the semigroup has generater A with matrix a

 $a(i,j) = \frac{d}{dt} \Big|_{t=0^{\dagger}} \operatorname{qt}(i,j).$ 

## Pf we proved last lecture that 11 A - Rt-Illop > 0 as the.

Take any bills

... By our proposition, A has matrix



Cor:  $a(i,j) \ge 0$  for  $i \neq j$ , and  $\sum_{j \in S} a(i,j) = 0$   $\forall i \in S$ 

(Exactly the same as the proof in [Lec. 39.2].)

