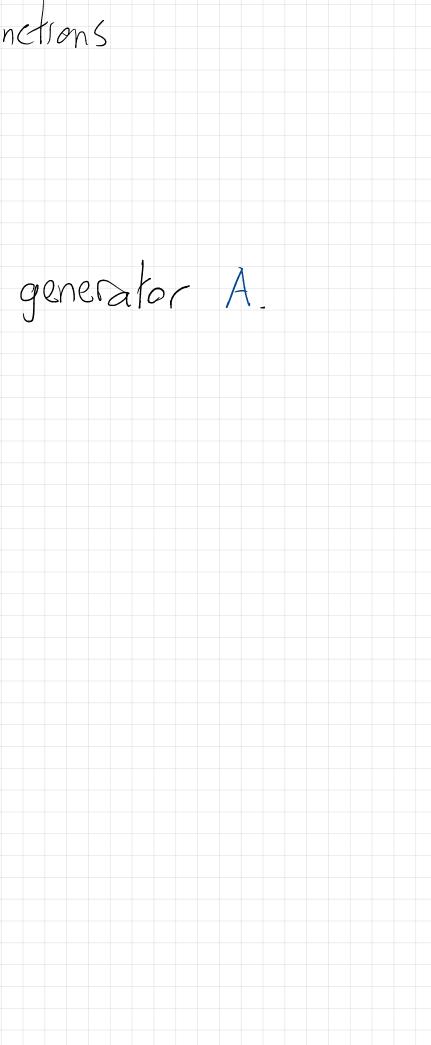


Thus, if (g.) we Markov transition kernel mass functions on a discrete state space S, satisfying $\lim_{t \to 1} \inf_{i \in S} G_t(i, i) = 1$ then the transition semigroup (Qt) too has a bounded generator A. Question: What can we say about A? Qt= etA $Af(i) = \frac{d}{dt}\Big|_{t=0^+} Q_t f(i)$ $= \frac{d}{dt} \left[\frac{d}{dzo^{T}} \sum_{j \in S} q_{j} (j) f(j) \right]$ $= \sum_{j \in S} d_{i} d_{j} d_{j$ This suggests that A has a matrix a given by $a(i,j) = \frac{d}{dt} \left(\frac{1}{t} \left(i, j \right) \right)_{t=0^+}$ If S is infinite, this takes some work to prove.



First question; does A even have a matrix?

You might think this must gloways be true: that every (bounded) linear operator

$T: B(S) \rightarrow B(S)$

has a matrix. Following the finite-dimensional case, we would expand

If f is a simple function, so f(j)= for all but finitely many jes,

 $Tf(i) = \sum_{i \in S} f(i) T(1_{Si}) (i)$

 $f = \sum_{i \in S} f(i) \mathbb{1}_{i}$

So we would expect that T has matrix $\Theta(i,j) = T(J(j))(i)$.

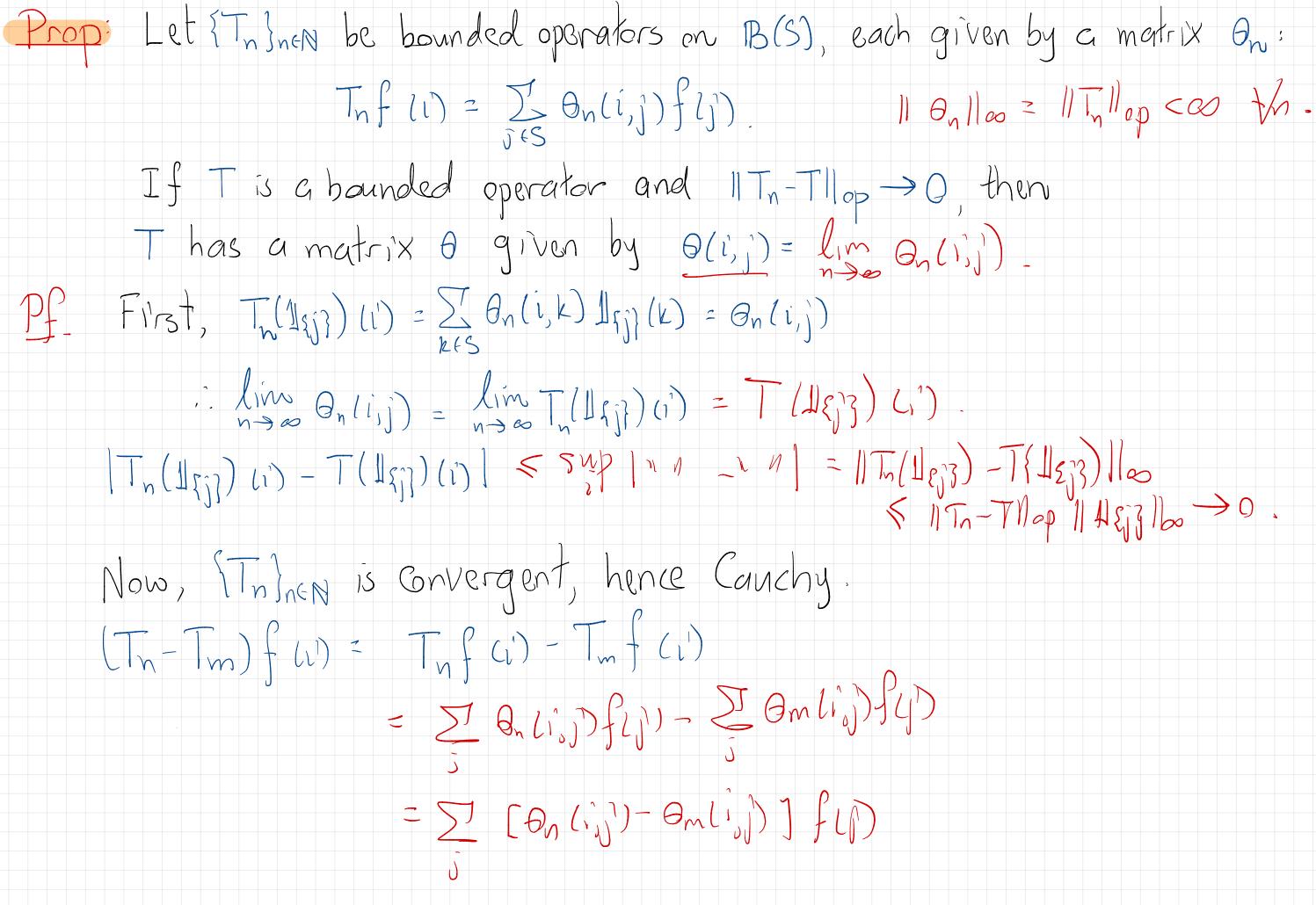
But if f is not simple, there's no way to extend this:

even if $\|\theta\|_{\infty} < \infty$, we can't check if $Tf(i) = \sum_{i} \theta(i,j)f(j)$.

Basic Problem: $f \equiv 1$ on \mathbb{N} . $\|f - f_{k}\|_{\infty} = 1$. $f_{k} = J_{\{1,2,\dots,k\}}$

Fact: I bounded operators on B(S) that have no matrix.





i. The has matrix on-on, so $\|\theta_n - \theta_n\|_{\infty} = \|T_n - T_n\|_{op} \to 0$ as n, m-200. $\frac{1}{2}\sum_{j} \frac{1}{2} \frac{1}{2}$ As this is true for all i, taking sup : 110-0110 5 11 Th-Tillop . By the triangle inequality, 1101100 < 00, so it defines a bounded operator $Tf(i) := \sum \theta(i,j)f(j).$ But then II T-Tullop = 110-001100 \$ 11 T-Tullop > 0 Since II T-Tullop > 0, it follows that T2T. $Te Tf(n) = Z \theta(n, j) f(j),$

Cor: Under the continuity Condition 11 Qi-Illop 70 is infquin-1 as the, the semigroup has generater A with matrix a $a(i,j) = \frac{q_1}{dt} + \frac{q_2}{t^2 - o^T} + \frac{q_1}{t} + \frac{q_2}{t} + \frac{q_1}{t^2} + \frac{q_2}{t} + \frac{q_1}{t^2} + \frac{q_2}{t^2} + \frac{q_1}{t^2} + \frac{q_1}{t^2} + \frac{q_2}{t^2} + \frac{q_1}{t^2} + \frac{q_1}{t^2} + \frac{q_2}{t^2} + \frac{q_1}{t^2} + \frac{q_1}{t^2} + \frac{q_1}{t^2} + \frac{q_2}{t^2} + \frac{q_1}{t^2} + \frac{q_1}$ Pf we proved last lecture that 11A - Rt-Illop > 0 as the. Take any billy in 11A - Qt-Illop 20 has maders to [genlin) - Sij]. .'. By our proposition, A has matrix $a(i,j) = \lim_{n \to \infty} \frac{1}{n} \left[\frac{q_{in}(i,j)}{q_{in}(i,j)} - \frac{1}{2n} \frac{q_{in}(i,j)}{q_{in}(i,j)} - \frac{1}{2n} \frac{q_{in}(i,j)}{q_{in}(i,j)} \right]$ $= \frac{d}{dt} \left[\frac{1}{t-0} - \frac{1}{2} \left[\frac{1}{t-0} \right] \right]$ Cor: $a(i,j) \ge 0$ for $i \neq j$, and $\sum_{j \in S} a(i,j) = 0$ $\forall i \in S$ (Exactly the same as the proof in [Lec. 39.2].)

